Supplemental Material

^{138}Ba^{+} Qubit State Detection

The details of the implemented ^{138}Ba^{+} qubit state detection technique are shown in Fig. 1. We collect ^{138}Ba^{+} fluorescence with a highly efficient optical objective (NA=0.6, or about 10% collection) and use an avalanche photodiode with 80% detection efficiency. This amounts to an effective 8% single-shot detection efficiency of the ^{138}Ba^{+} qubit. With such low detection efficiency, it is important to minimize sensitivity to slow drifts in photon collection efficiency or the excitation laser. We therefore collect fluorescence after hundreds of repeated runs of the same experiment by alternating the sense of circular polarization of the excitation laser. In this way, we detect both ^{138}Ba^{+} qubit states independently, providing normalization for the extracted population probability. Moreover, this technique suppresses statistical bias in the photon collection distribution between the $|\downarrow\rangle$ and $|\uparrow\rangle$ states.

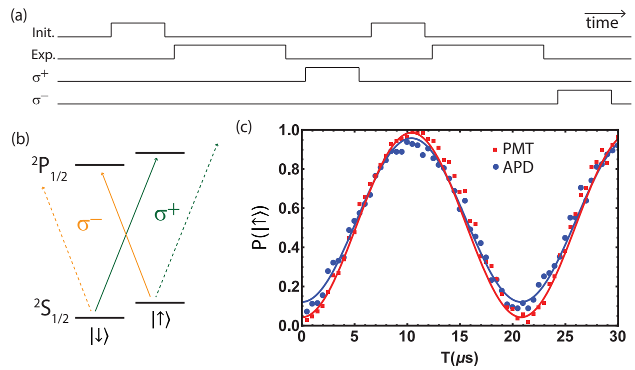

FIG. 1. (color online) (a) ^{138}Ba^{+} detection timing sequence. After initializing to $|\downarrow\rangle$ and preparing the ^{138}Ba^{+} qubit (an "experiment"), we excite the $^2S_{1/2}$ to $^2P_{1/2}$ transition with 493 nm σ^+ polarized light. If the ion is in the $|\downarrow\rangle$ state, on average it scatters 3 photons before getting optically pumped to the $|\uparrow\rangle$ state, and each photon is detected with a probability of approximately 8%. In contrast, the $|\uparrow\rangle$ state ideally does not scatter any photons due to selection rules. We then repeat the same initialization/experiment steps, but now send σ^- polarized light, and the situation is reversed with only the $|\uparrow\rangle$ state scattering photons. We cycle between σ^+ and σ^- detection on identical experiments hundreds of times, and accumulate the collected photons for each detection polarization condition. This allows us to infer the ^{138}Ba^{+} qubit state without statistical bias or drifts. (b) Relevant energy levels in the ^{138}Ba^{+} system coupled with σ^+ and σ^- resonance excitation light at 493 nm. (c) Observed Rabi oscillations between the $|\downarrow\rangle$ and $|\uparrow\rangle$ states, where the experiment consists of stimulated optical Raman transtions between the states with variable duration T. The probability of finding the ^{138}Ba^{+} qubit in $|\uparrow\rangle$ is extracted from the ratio of the photons collected in the σ^- cycle to the total of 300 photons collected with either polarization. The total integration time for the entire 1.5 Rabi cycles is approximately 2 minutes. While the effective ^{138}Ba^{+} qubit detection efficiency is only 8%, we estimate a detection accuracy of about 94%. Using a photomultiplier tube (PMT) that has a lower dark count rate compared to the avalanche photodiode (APD), about 10 and 2000 cps respectively, we get a higher detection accuracy of about 98%. However, usage of a PMT results in a slower data acquisition speed due to lower efficiency of 30%.

Phase Considerations in the Quantum Networks

The full truth table of the Mølmer-Sørensen (MS) entangling gate is given by:

$$|\Downarrow\rangle|\downarrow\rangle \to \frac{1}{\sqrt{2}}(|\Downarrow\rangle|\downarrow\rangle - ie^{i(\phi_{S,1}+\phi_{S,2})}|\Uparrow\rangle|\uparrow\rangle)$$

$$|\Downarrow\rangle|\uparrow\rangle \to \frac{1}{\sqrt{2}}(|\Downarrow\rangle|\uparrow\rangle - i|\Uparrow\rangle|\downarrow\rangle)$$

$$|\Uparrow\rangle|\downarrow\rangle \to \frac{1}{\sqrt{2}}(|\Uparrow\rangle|\downarrow\rangle - i|\Downarrow\rangle|\uparrow\rangle)$$

$$|\Uparrow\rangle|\uparrow\rangle \to \frac{1}{\sqrt{2}}(|\Uparrow\rangle|\uparrow\rangle - ie^{-i(\phi_{S,1}+\phi_{S,2})}|\Downarrow\rangle|\downarrow\rangle)$$

where the spin phases $\phi_{S,i}$ are dependent on the optical path lengths and they are written directly to the entangled state. In a multi-species quantum network, the aim is transferring a superposition state in ^{138}Ba^{+} , $\alpha|\downarrow\rangle + e^{i\phi}|\uparrow\rangle$, to the memory ^{171}Yb^{+} qubit, $|\Downarrow\rangle$. The method demonstrated in Ref. [35] to resolve optical phase issues is not directly compatible with a multi-species setup since canceling the ϕ_S optical sensitivity transfers this effect to the phase of the applied force which would result in variance of phase space trajectories and the accumulated geometric phase. Instead, extra single qubit rotations can be included to cancel any optical path dependence of spin phases [15,34]. However, without resorting to these techniques, applying two MS gates to the joint state with a π phase shift on the total spin phase in the second operation with respect to the first, results in transfer of the ^{138}Ba^{+} superposition to ^{171}Yb^{+} without introducing extra optical phases on the final state:

$$U_{\rm MS}(\phi_{S,1}-\pi,\phi_{S,2})U_{\rm MS}(\phi_{S,1},\phi_{S,2})[|\Downarrow\rangle(\alpha|\downarrow\rangle + \beta e^{i\phi}|\uparrow\rangle)] \to |\downarrow\rangle(\alpha|\Downarrow\rangle - i\beta e^{i\phi}|\Uparrow\rangle)$$

The required π spin phase shift can be realized by advancing the red and blue sideband RF phases by π for the second MS operation. More experimental details about this setup are given in Ref. [33].