12.13. In Minor's rest frame, incoming light has Doppler shift:

\[ f_1 = f_0 \sqrt{\frac{c+v}{c-v}} \]

After reflection, light has \( f_1 \) in mirror's frame. Since mirror is approaching the observer with \( v \), so there is a second Doppler shift:

\[ f_2 = f_1 \sqrt{\frac{c+v}{c-v}} = f_0 \sqrt{\frac{(c+v)^2}{(c-v)^2}} = f_0 \left( \frac{c+v}{c-v} \right) \]

We can think of the total effect coming from the image source. From Lorentz transformation, we can find the velocity of the image in observer's frame:

\[ V' = \frac{V + V}{1 + \frac{V^2}{c^2}} = \frac{2V}{1 + \frac{V^2}{c^2}} \]

We can check the Doppler shift from the image is:

\[ f' = f_0 \sqrt{\frac{c+V'}{c-V'}} = f_0 \sqrt{\frac{(c+V')^2}{(c-V')^2}} = f_2 \]

This matches previous result.
12.14. In A. B’s rest frame, we can find:

Speed of light in glass \( V_{in} = \frac{c_n + V}{1 + \frac{c_n V}{c^2}} \)

Length of glass slab \( D' = D \sqrt{1 - \frac{V^2}{c^2}} \)

Because glass slab is moving in the same direction of light, so the actual time that light is in glass \((t_{in})\) and the total distance that light travels in glass \((L_{in})\) are:

\[
\begin{align*}
    t_{in} &= \frac{D'}{V_{in} - V} = D \frac{(1 + \frac{V}{c_n})}{\frac{c}{n}} \frac{1}{\sqrt{1 - \frac{V^2}{c^2}}} \\
    L_{in} &= t_{in} V_{in} = D \frac{(c_n + V)}{\frac{c}{n}} \frac{1}{\sqrt{1 - \frac{V^2}{c^2}}}
\end{align*}
\]

Since \( L = L_{in} + L_{out} \), we can find

\[
L_{out} = L - D \frac{(c_n + V)}{\frac{c}{n}} \frac{1}{\sqrt{1 - \frac{V^2}{c^2}}}
\]

Total time from A to B is:

\[
\begin{align*}
    t &= t_{in} + \frac{L_{out}}{c} \\
    &= \frac{L}{c} + \frac{D}{c} \frac{1}{\sqrt{1 - \frac{V^2}{c^2}}} \left( n + \frac{V}{c} - 1 - \frac{V^2}{c^2} \right) \\
    &= \frac{L}{c} + \frac{D}{c} (n-1) \sqrt{\frac{c-V}{c+V}}
\end{align*}
\]
(a) \( C = \lambda \cdot f \)

Doppler effect for wavelength:

\[ \lambda = \lambda_0 \sqrt{\frac{C + V}{C - V}} \]

\[ \lambda_0 = 6.561 \times 10^7 \text{ m} \quad V = 3 \times 10^6 \text{ m/s} \]

So:

\[ \lambda = 6.627 \times 10^7 \text{ m} \]

(b) \( \lambda_1 \) and \( \lambda_2 \) are wavelength from two ends of the sun.

\[ \lambda_1 = \lambda_0 \sqrt{\frac{C + V'}{C - V'}} \]

\[ \lambda_2 = \lambda_0 \sqrt{\frac{C - V'}{C + V'}} \]

\[ \Delta \lambda = \lambda_0 \left( \sqrt{\frac{C + V'}{C - V'}} - \sqrt{\frac{C - V'}{C + V'}} \right) \]

\[ \approx \lambda_0 \left( 1 + \frac{V'}{C} \right) - \left( 1 - \frac{V'}{C} \right) \]

\[ \Delta \lambda \approx \lambda_0 \frac{V'}{C} \]

\[ V' = \frac{\Delta \lambda}{2 \lambda_0} \cdot C = \frac{9 \times 10^{12}}{2 \times 6.561 \times 10^7} \quad 3 \times 10^6 \text{ m/s} \approx 2.06 \times 10^3 \text{ m/s} \]

Period:

\[ T = \frac{\pi D}{V'} = \frac{\pi \times 1.4 \times 10^9 \text{ m}}{2.06 \times 10^3 \text{ m/s}} \approx 2.13 \times 10^6 \text{ s} \]

\[ \approx 24.7 \text{ days.} \]
12.16 (a). From length contraction:

\[ l = l_0 \sqrt{1 - \frac{v^2}{c^2}} = \frac{1}{2} l_0 \]

(b) Length of the barn in \( s' \) frame is:

\[ l_{\text{bam}} = l_{\text{bam}}' \sqrt{1 - \frac{v^2}{c^2}} = \frac{3}{4} l_0 \times \frac{1}{2} = \frac{3}{8} l_0 \]

\[ \Delta l = l_0 - l_{\text{bam}}' = \frac{5}{8} l_0 \]

So in \( s' \) frame, B is \( \frac{5}{8} l_0 \) away from the front door.

(c). Because \( l_0 > l_{\text{bam}}' \) in \( s' \) frame, so A, B do not lie inside the barn at the same instant.