Realization of a Filter with Helical Components*

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Summary—**In the VHF range, high-quality narrow-band filters with reasonable physical dimensions are extremely difficult to realize. Excessive pass-band insertion loss accompanies filters employing lumped constant elements, and unreasonable size is a natural consequence of coaxial-resonator filters. Harmonic crystal filters are inadequate because of the unpredictable amount of spurious modes above the harmonic frequency; and hence they can only be used for very low percentage bandwidths. A solution to the above problems is provided by the use of helical resonators for high-quality filters.**

This paper describes the helical resonator, measurement of its *Q*, and coupling to the resonator. A procedure for the design and construction of a filter using helical circuits is then presented. Finally an example is given illustrating the design method, and several photographs of helical-resonator filters are shown.

I. Introduction

IT is a well-established fact that in the VHF range high-selectivity filters are extremely difficult to realize. High-quality, practical-sized lumped elements cannot be successfully used at frequencies above approximately 30 Mc, and coaxial resonators in the VHF range become large and cumbersome. Crystal filters, which are very popular at the present time, cannot be used to solve the majority of VHF filter problems because of limitations on their realization. Let us recollect the known difficulties of filtering in the above-mentioned frequency range.

All filters require a certain minimum value for the quality factor of the resonator or lumped reactance. Inductors in toroidal form are not generally used above 30 Mc because of distributed capacitance effect, and shielded single-layer solenoids of reasonable size will only provide a maximum Q of about 200. In order to increase the Q factor, the size of the coil must be increased to unreasonable proportions. Even when the coil is made large, the Q is still relatively low and the space requirements will naturally not be satisfied. The Q factor of coaxial resonators is usually very high, but in the VHF range their construction becomes bulky and unproportionally large in comparison with the size of present tubes, resistors, capacitors, and other components of modern circuitry.

The piezo-electric crystal resonator is the only component which satisfies the electrical requirement of high Q and the physical requirement of small size. However, at the present time, it is practically impossible to construct crystals whose fundamental frequency is above 35 Mc. Their internal physical dimensions become so small that a good quality crystal cannot be produced. Harmonic crystals are generally utilized at frequencies above 30 Mc. The crystal itself is basically a very high-quality device, but it inherently possesses several shortcomings which appreciably reduce the domain of its utilization for filter construction. Crystals always have an unpredictable amount of spurious modes above the fundamental frequency (or any harmonic frequency), and hence they can only be used for very low percentage bandwidths (below 1 per cent). Any attempt to create wide-band crystal filters meets unsurpassable difficulties becuase of the necessity to include lossy spreading coils, which reduce the obtainable bandwidth from its theoretical maximum value. A crystal filter is basically a very narrow bandpass element.

For a long time the practicing engineer has looked for a new type of resonator which would provide a solution to the seemingly unsolvable problem of filtering in the VHF domain. The solution has come from an adjacent side of electronics—antenna design. We refer to the coaxial line with the helical inner conductor. Coaxial lines with helical inner conductors are used in traveling-wave tubes, as delay lines, high-Q resonators, high characteristic impedance transmission lines, and in extending impedance-matching techniques to frequencies as low as 300 kc. As yet they have not been used in filters.

Results of extensive experimentation in the use of helical resonators for high-quality filters in the VHF range will be given here.

II. Helical Resonator

Helical resonators of practical size and form factor, and with high Q (greater than 1000) can be constructed for the VHF range. Basically they resemble a coaxial quarter-wave resonator, except that the inner conductor is in the form of a single-layer solenoid (also called a helix). It is only natural to suspect that resonators of this form can be extended to higher frequencies, when a Q between several hundred and 1000 is sufficient.

The helical resonator consists of a single-layer solenoid enclosed in a shield made from a highly conductive material. The shield may have a circular or square cross section. One lead of the helical winding is connected directly to the shield and the other lead is open circuited.

As an example of space saving and superior form factor, consider a coaxial resonator at 54 Mc with an unloaded Q of 550. The coaxial resonator would be 4.5 feet long by 0.7 inches in diameter. The same quality helical resonator would be 1.5 inches in diameter and 2 inches long.

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A. Design

Fig. 1 shows a sketch of the resonator with circular cross section. With these notations, the following set of equations,[1] which are well known to microwave specialists, can be given:

$$L = 0.025n^2 d^2[1 - (d/D)^2]\mu\text{h/axial inch}, \tag{1}$$

where

L is the equivalent inductance of the resonator in μh per axial inch
d is the mean diameter of the turns in inches
D is the inside diameter of the shield in inches

$$n = \frac{1}{\tau} = \text{turns/inch}, \tag{2}$$

where

τ is the pitch of the winding in inches.
Empirically for air dielectric,

$$C = \frac{0.75}{\log_{10}(D/d)}\ \mu\mu\text{f/axial inch}. \tag{3}$$

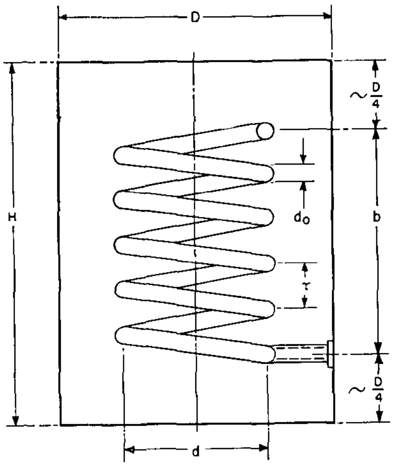

Fig. 1—Helical resonator.

This equation is only valid for the following condition:

$$\frac{b}{d} = 1.5, \tag{3}$$

where

b is the axial length of the coil in inches.

These equations and all those below are accurate for the resonator when it is realized between the following limits:

$1.0 < b/d < 4.0$

$0.45 < d/D < 0.6$

$0.4 < d_0/\tau < 0.6$ at $b/d = 1.5$

$0.5 < d_0/\tau < 0.7$ at $b/d = 4.0$

$\tau < d/2,$

where

d_0 is the diameter of the conductor in inches.

The axial length of the coil is approximately equivalent to a quarter wavelength. This actual length is much shorter than the free-space length, which is given by the expression

$$\frac{\lambda}{4} = \frac{c}{4f_0} \times 10^{-6}, \tag{4}$$

where

c is the speed of light in free space

and

f_0 is the operating frequency in Mc.

The actual length of the coil in inches can be expressed by the following equation:

$$b = \frac{250}{f_0\sqrt{LC}}, \tag{5}$$

where

f_0 is the resonant frequency in Mc.

This expression is based on theoretical considerations, but a working equation can be formulated with the help of the following expression:

$$\text{wave velocity,}\quad v = f_0\lambda = \frac{2\pi\,\text{RAD}}{2\pi\sqrt{LC}} = \frac{1000}{\sqrt{LC}}. \tag{6}$$

Because of fringing effect and self-capacitance of the coil, the electrical length of the coil is approximately 6 per cent less than a quarter wavelength. The empirical value of b will be reduced by 6 per cent and is

$$b = \frac{0.94\lambda}{4} = \frac{0.235v}{f_0} = \frac{235}{f_0\sqrt{LC}}. \tag{7}$$

The number of turns per inch is obtained by substituting (1) and (3) into (5).

$$\frac{1}{\tau} = n = \frac{1720}{f_0\, D^2(b/d)(d/D)^2}\left[\frac{\log_{10}(D/d)}{1-(d/D)^2}\right]^{1/2} \text{turns per inch.} \tag{8}$$

The total number of turns N is given by

$$N = nb = \frac{1720}{f_0\, D(d/D)}\left[\frac{\log_{10}(D/d)}{1-(d/D)^2}\right]^{1/2} \text{turns.} \tag{9}$$

The characteristic impedance of the resonator is expressed by

[1] W. W. Macalpine and R. O. Schildknecht, "Coaxial resonators with helical inner conductor," Proc. IRE, vol. 47, pp. 2099–2105; December, 1959.

$$Z_0 = \frac{1000}{\sqrt{LC}} = 183nd\{[1 - (d/D)^2] \log_{10} (D/d)\}^{1/2} \text{ ohms.} \tag{10}$$

If

$$\frac{d}{D} = 0.55 \quad \text{and} \quad \frac{b}{d} = 1.5,$$

then

$$N = \frac{1900}{f_0 D} \text{ turns,} \tag{11}$$

and

$$Z_0 = \frac{98000}{f_0 D} \text{ ohms.} \tag{12}$$

If the shield is of square cross section, the following equations are applicable:

$$S = \text{length of one side of the square} = \frac{D}{1.2}$$

$$Q = 60S\sqrt{f_0} \tag{13}$$

$$N = \frac{1600}{f_0 S} \tag{14}$$

$$n = \frac{1}{\tau} = \frac{1600}{S^2 f_0} \tag{15}$$

$$Z_0 = \frac{81500}{f_0 S} \tag{16}$$

$$d = 0.66S \quad \text{for} \quad d/D = 0.55 \tag{17}$$

$$b = S \quad \text{for} \quad b/d = 1.5 \tag{18}$$

$$H = 1.6S. \tag{19}$$

Fig. 2 shows the nomogram constructed from formulas (13)–(19). This nomogram is to be used for helical resonators in shields of square cross section, the resonator that physically lends itself best to filter design.

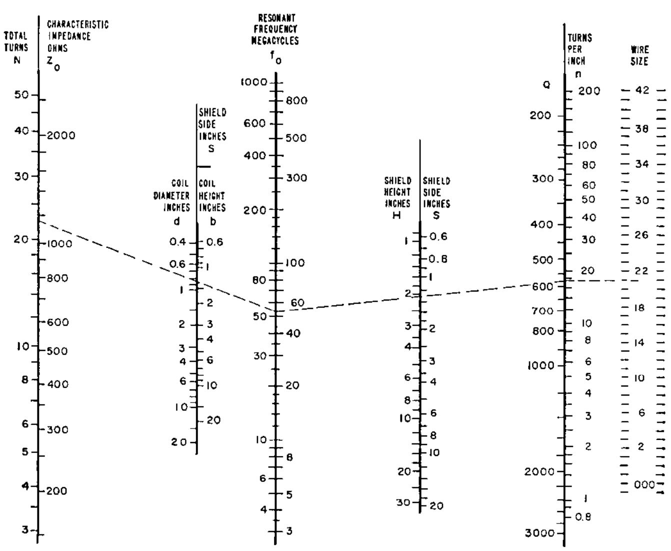

Fig. 2—Nomogram for helical resonators in shields of square cross section.

B. Quality Factor

As mentioned before, the helical resonator solves the immediate problem of high-quality resonators in the VHF range, the "no man's land" where all conventional resonators fail to meet the basic requirements for filter applications. In a reasonable volume they provide a tuned circuit whose Q is higher than a normal lumped circuit. Possible causes of dissipation in the resonator are losses in the conductor, the windings, the shield, and the dielectric.

The Q of the resonator—electrical or mechanical—is defined by

$$\begin{aligned} Q &= 2\pi \frac{\text{(energy stored)}}{\text{(energy dissipated per cycle)}} \\ &= 2\pi f \frac{\text{(energy stored)}}{\text{(power dissipation)}}. \end{aligned} \tag{20}$$

The most important loss in coils is the copper loss, as influenced by skin and proximity effect. There is also an additional loss due to currents in the shield. The resistance of the coil can be expressed as

$$R_c = \frac{n\pi\, d\phi \sqrt{f}}{12000\, d_0},$$

or,

$$R_c = \frac{0.083}{1000} \cdot \frac{\phi}{nd_0} \cdot n^2 \pi\, d\sqrt{f}\ \Omega/\text{axial inch.} \tag{21}$$

An additional resistance due to the shield is given by

$$R_s = \frac{9.37n^2 b^2 (d/2)^4 \sqrt{1.724f}}{b[D^2(b + d)/8]^{4/3}} \sqrt{\frac{\rho_s}{\rho_{cu}}} \times 10^{-4}\Omega/\text{axial inch.} \tag{22}$$

The unloaded Q of a resonant line[2] is given by

$$Q_u = \frac{\beta}{2\alpha}. \tag{23}$$

If R_c and R_s are assumed in series, the Q of the resonant line is expressed as

$$Q_u = \frac{2\pi f_0 L}{R_s + R_c}. \tag{24}$$

In this form, the dielectric losses are neglected. For a resonator with a copper coil and copper shield, (21) and (22) can be substituted into (24), and the final expression for the unloaded Q[3] is

$$Q_u = 220 \frac{(d/D) - (d/D)^3}{1.5 + (d/D)^3} D\sqrt{f_0} \tag{25}$$

$$\approx 50\, D\sqrt{f_0}. \tag{26}$$

[2] "Reference Data for Radio Engineers," ITT Corp., New York, N. Y., 4th ed., p. 575; 1956.
[3] *Ibid*, p. 602.

This simplified equation is accurate to ±10 per cent and is derived with three practical limitations:

$$0.45 < \frac{d}{D} < 0.6,$$

$$\frac{b}{d} > 1.0,$$

and

$$d_0 > 5\ \delta,$$

where

δ is the skin depth.

For copper conductors[4]

$$\delta = \frac{2.60 \times 10^{-3}}{\sqrt{f_0}} \text{ inch.} \tag{27}$$

To show how important the volume of the resonator is, the Q_u as a function of volume is

$$Q_u = 50\sqrt[3]{\text{vol}} \cdot \sqrt{f_0},$$

when

$$0.4 < \frac{d}{D} < 0.6 \quad \text{and} \quad 1 < \frac{b}{d} < 3. \tag{28}$$

Fig. 3 illustrates how rapidly the unloaded Q decreases as a function of d/D and how important it is to keep this ratio between the specified limits.

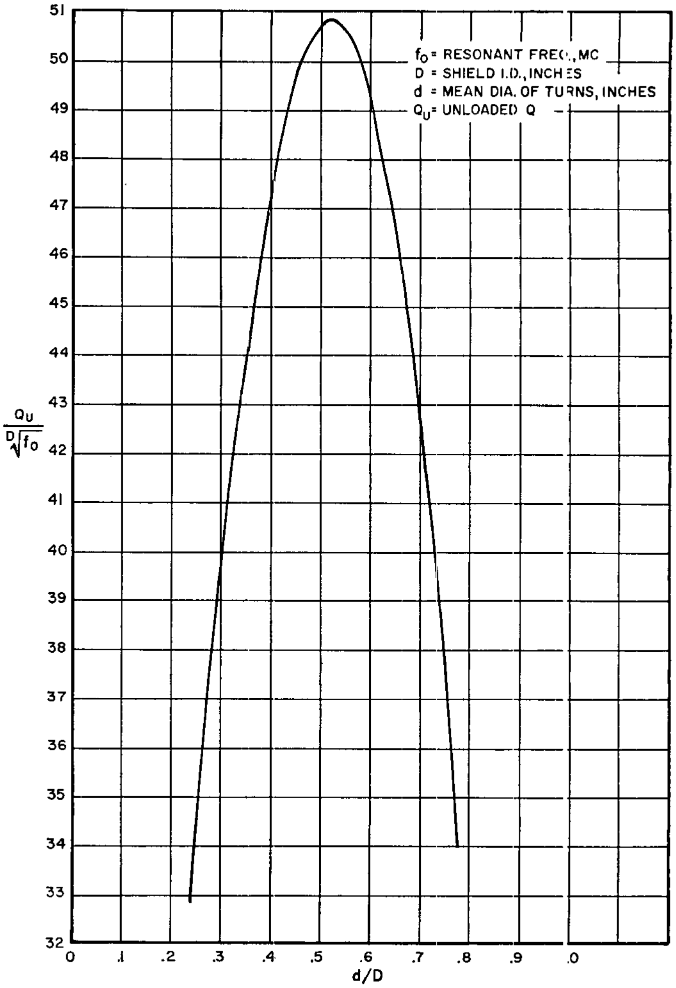

Fig. 3—Unloaded Q of the helical resonator.

C. Measurement of Resonator Q

The problem of finding the unloaded Q of the resonator is not easy. Many methods have been proposed, but most have been inconvenient or impractical. However, the unloaded Q can be estimated quite accurately from the loaded Q and the insertion loss.[5] This relation between the insertion loss and Q when generator and load impedances are equal is

$$L_{\text{DB}} = 20 \log \frac{U}{U-1},$$

where

$$U = \frac{Q_{\text{unloaded}}}{Q_{\text{minimum}}}. \tag{29}$$

In this case, $Q_{\min}$ is the loaded Q determined from the relation $Q_{\min} = f_0/(BW)_{3\text{db}}$, where $(BW)_{3\text{db}}$ is the measured 3-db bandwidth. When loop coupling is used into and out of the resonator the insertion loss, and hence $Q_{\min}$, will be a function of the coupling between loops and the losses in the loop circuits. It is desirable to use very loose coupling in order that the effect of coupling between loops may be neglected. Fig. 4 gives a plot of (29). The insertion loss is measured by the substitution method when all coaxial cables are as short as possible. The value of unloaded Q is evaluated by multiplying the value of $Q_{\min}$ by the value of U, which corresponds to the measured insertion loss. If the insertion loss of the resonator is greater than 25 db, the correction factor for the unloaded Q will be 1.05 or less. At this condition, $Q_{\min}$ will only be in error of Q_{unloaded} by 5 per cent and it is self-evident that Q_{unloaded} could be calculated from $f_0/(BW)_{3\text{db}}$.

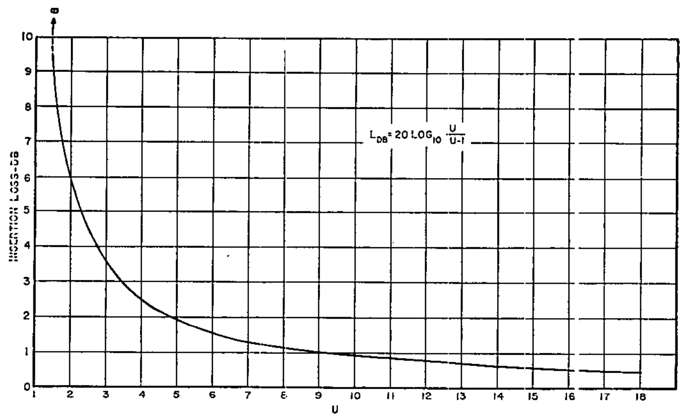

Fig. 4—General curve for minimum insertion loss.

[4] *Ibid.*, p. 129.
[5] E. G. Fubini and E. A. Guillemin, "Minimum insertion loss filters," Proc. IRE, vol. 47, pp. 37–41; January, 1959.

D. Physical Construction of Resonator

To obtain the predicted unloaded Q, several important points for construction of the resonator should be remembered. The coil form should be made of a low-loss material such as teflon or polystyrene. If the dielectric material is sufficiently rigid, the coil form can be nothing more than a hollow cylinder. Helices using larger wire sizes may not require a winding form and can be self supported by the connected end. Prewound air-core coils can also be used. It is desirable to silver plate the surface of the wire and the shield to increase the conductivity. The form of the shield can be cylindrical, rectangular, or any other shape, but for simplicity of calculations only a shield with a circular or square cross section has been considered. There should be no seam in the shield parallel to the coil axis. The lower end of the coil should be carried over and soldered (or welded) to the shield as directly as possible. If the coil end is run to the bottom cover of the shield, the cover must be solidly soldered to the shield in order to reduce the losses in the joints. The length of the shield must be extended beyond the coil on each side by approximately one quarter of the shield diameter. If the coil were carried to the bottom of the shield without having this clearance, the lower few turns would be ineffective for storage of energy but would still contribute loss. The clearance, at the top of the resonator, reduces capacitive loading due to fringing.

The resonator can be open since the top and bottom shield have little effect upon the frequency and Q. The external field is minimized by use of the top and bottom covers.

E. Coupling

A very important problem is now considered—that of coupling into and out of the helical resonators. Several ways to effectively couple to the helical resonator are loop coupling at the bottom of the coil, probe coupling at the top of the resonator, or aperture coupling in a manner analogus to direct-coupled coaxial resonators.

A loop for low-impedance coupling is usually placed below the actual coil in a plane perpendicular to the coil axis and only a small distance from the coil itself. In the case of a filter, it is only natural to suspect that the distance between the coupling loop and helix can be used for adjustment of the standing wave ratio or echo attenuation.

For high-impedance coupling between input and output circuit and resonator, or between resonators, the probe is placed close to the upper part of the helix. In this manner no direct current circuits are provided and the coupling is mostly capacitive. With small capacitors as the coupling elements between resonators, an appreciable amount of coupling, which determines the bandwidth, can be obtained. Openings between the resonators can also be used for coupling. Openings at the upper part are equivalent to capacitive coupling; that is, more attenuation is obtained on the lower side of the pass band than on the upper. Openings in the lower part of the partitions provide inductive coupling, and the filter will exhibit more attenuation on the high side than on the low side.

III. Filter with Helical Resonators

To develop a filter, it is necessary to prescribe the desired effective attenuation as a function of frequency. In most instances, this requirement is simple. In the pass band, the attenuation must be small, while in the stop band, the attenuation must be greater than a given value. Between these two bands lies the transition band where the attenuation rises from a small to a large value. The problem is to realize a transmission function which satisfies the above attenuation requirements.

A filter with the above requirements can be realized by two different kinds of responses: Butterworth and Tchebycheff. The first type of response, which gives no ripples in the pass band, may be considered as the limiting case of the more general Tchebycheff response. With the response, often called a maximally flat characteristic, minimum attenuation in the pass band is obtained. A Tchebycheff response is one with maximum slope in the transition region between pass and stop band. The attenuation vs frequency is allowed to oscillate or ripple between prescribed limits in the pass band. In general, the bigger the ripples in the pass band, the steeper the slope of attenuation in the transition region.

A. Determination of Q Factor

For a given set of specifications, the value of the unloaded Q must exceed a certain Q_{min} in order that the filter be realizable. Fig. 5[6] shows the relationship between the required $q_{min}(Q_{min} = q_{min} f_0/\Delta f)$ for a Butterworth and three different Tchebycheff filters.

Example: A 7-pole filter (7 resonators) possessing a Butterworth response requires a q_{min} of 4.6. For a Tchebycheff response with a 1-db ripple in the pass band, $q_{min} = 21.9$. If Q_{unl} were equal to Q_{min}, the insertion loss would be infinite. It is self evident that the unloaded Q must exceed Q_{min}. The values of q_{min} have been tabulated for both the Butterworth and Tchebycheff cases and the theoretical performance of these filters is shown.[7]

For the above cases, assuming a 1 per cent bandwidth, Q_{min} equals 460 for the Butterworth filter and 2190 for the Tchebycheff. It must be remembered that if components, whose unloaded Q is barely equal to Q_{min}, are used, the response can be achieved, but an infinite midband loss will result.

When the unloaded Q is greater than Q_{min}, the loss of the filter does not primarily depend on the number of sections, but is exclusively controlled by the ratio U, given in (29). Once the minimum value of unloaded Q is obtained and the quality factor of available components

[6] *Ibid.*, p. 37.
[7] "Reference Data for Radio Engineers," *op. cit.*, pp. 193–198.

is determined, the loss in the filter is almost completely defined and varies very little with the shape of the filter, the number of sections, the bandwidth, etc.

Fubini and Guillemin[8] give a curve that shows the minimum insertion loss at midband of Butterworth filters plotted as a function of the ratio U. The following two conclusions can be made.

1) For moderate losses, the curves are very close to each other,
2) The curves for 1 and 2 section filters are exactly the same and are expressed by (29).

From the previous example of a 7-pole Butterworth filter, assume the available unloaded Q of each section is 3000. The value of U can be computed as follows:

$$U = \frac{Q_{\text{unl}}}{Q_{\text{min}}} = \frac{3000}{460} = 6.54.$$

From (29) or Fig. 4,

$$L_{\text{DB}} = 20 \log \frac{6.54}{5.54} = 20 \times 0.071 = 1.42 \text{ db}.$$

As mentioned before, this equation is only valid for 1 and 2 sections. For the example with 7 resonators, a correction factor must be used. Fig. 6 plots this correction factor given by Fubini and Guillemin and shows that the loss is always greater as the number of section is increased.

Since the number of sections is 7, the correction factor is 1.27 and the actual insertion loss at midband will be

$$L_{\text{DB}} = 1.42 \times 1.27 = 1.8 \text{ db}.$$

For a realizable Tchebycheff filter, q_{min} is always higher than that of a Butterworth filter and can be found from Fig. 5. For the same unloaded Q and bandwidth, the insertion loss of a Tchebycheff filter will be several times higher than that of a Butterworth filter.

Example: If the available unloaded Q is 5000 and the relative bandwidth is 1 per cent, the insertion loss for a 4-section Butterworth filter will be 0.48 db. For a Tchebycheff filter with 4 sections (3-db ripple) the expected insertion loss will be 2.3 db. If the Q factor is only 3000, the former will result in 0.935-db loss and the latter will exhibit 4.3-db insertion loss. If the Q is 2000, the corresponding insertion losses will be 1.2 and 7 db, respectively. For more information concerning this subject, see the literature.[9]

B. Filter Construction

As previously stated, the only reason for using helical resonators is to reduce the size of the filter and to provide a low insertion loss in the pass band. The design of a filter for a Butterworth and Tchebycheff response is

[8] Fubini and Guillemin, *op. cit.*, p. 38.
[9] "Reference Data for Radio Engineers," *op. cit.*, pp. 187–235.

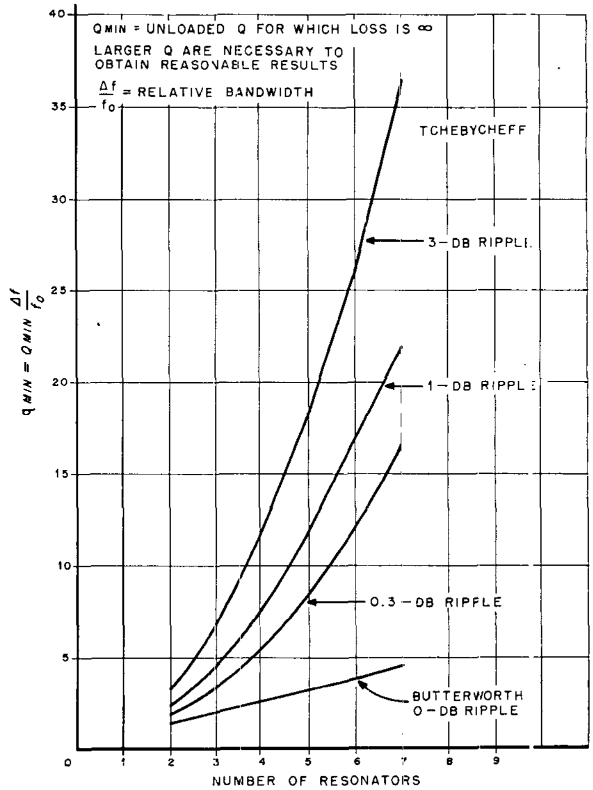

Fig. 5—Relative minimum unloaded Q for Butterworth and Tchebycheff filters.

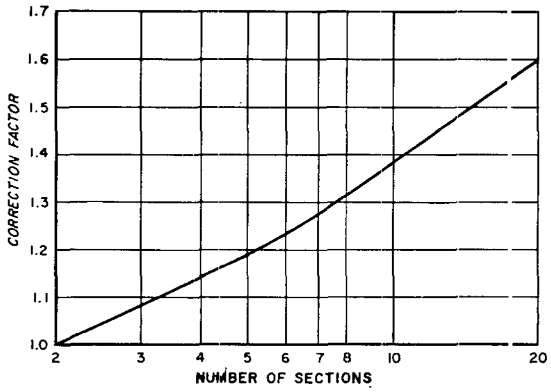

Fig. 6—Correction factor for insertion loss for two or more sections.

straightforward, and the evaluation of their circuit elements is well known. More interesting, however, are the equivalent schematic and the mechanical realization of the filter.

Even after carefully calculating the number of turns and all dimensions of the resonator, it is very possible that the resonant frequency may be in error by as much as 10 per cent. This must be adjusted without any distortion to the other dimensions so that the predicted Q will be obtained. This adjustment is made by using a brass screw at the top of the helix. In the equivalent schematic, the screw, because it is connected to the ground, has the effect of providing capacitive loading for the helix.

Fig. 7 shows the equivalent schematic of a 2-cavity filter when a lumped capacitance exists. Fig. 8 shows a 3-cavity filter based upon the same principle of a helical resonator inside each cavity and magnetic coupling between them through the openings. Coupling in and out of the filter is provided by low-impedance loops.

C. Coupling Between Resonators

The coupling of helical resonators is considered the most complicated problem of the filter design. However, the problem is greatly simplified if the coupling is mostly inductive. Fig. 9 shows a physical construction which provides this. The shield is made of the same material as that of the can and is solidly connected to the sides and top of the can (spot welded, dip brazed, or soldered). The dimension h determines the amount of coupling between resonators, and a method will be given here for calculating it.

The coupling is considered to be entirely inductive, a valid assumption for narrow-band filters (<8 per cent). The shield will then be far enough down the coil so effectively most of the capacitive coupling is eliminated. The calculation of the coupling between coils with parallel axis[10] has already been done, and with a slight modification it can be applied to helical resonators. One of the primary ratios to be considered is S/d. For the dimensions in (13)–(19), $S/d = 1.52$. Fig. 10 is based upon this value, and if the actual ratio is much different (>15 per cent), Dwight[10] should be consulted. The quantities shown on Fig. 10 will now be defined.

S, N, and d are given in (13) through (19).

h, from Fig. 9 is seen to be the distance from the bottom of the coil to the beginning of the shield.

M is the mutual inductance between coils. For small percentage bandwidth filters (<8 per cent), M can be given quite accurately by the relationship

$$M_{12} = k_{12} \times \frac{L}{2} \times \frac{(BW)_{3\text{db}}}{f_0}, \tag{30}$$

where the subscripts 1 and 2 refer to the first and second resonator, respectively. The mutual inductance between the 2nd and 3rd coil is given by M_{23}. The normalized coefficients of coupling can be found in the literature.[9]

The procedure for determining h is to first make an educated guess. Calculate h/d and enter Fig. 10 with this value. Three curves are given for different values of N. It can be shown[11] that the coupling is a function of the ratio d_0/d. By substitutions involving (14), (15), and (17), and the condition that

$$\frac{d_0}{\tau} = 0.5,$$

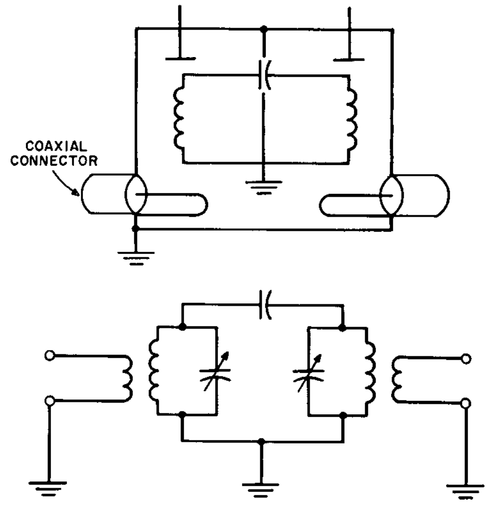

Fig. 7—Equivalent schematic of a 2-cavity filter.

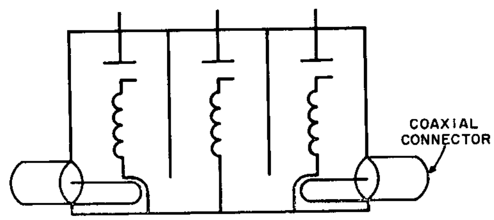

Fig. 8—A 3-cavity filter.

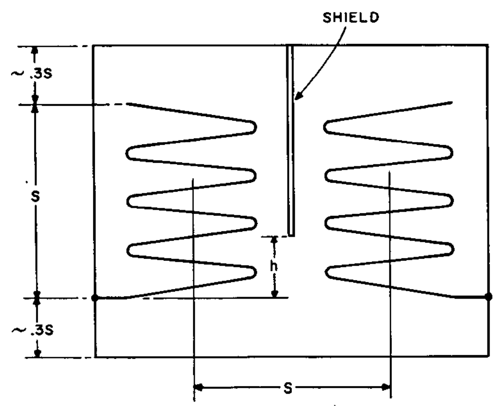

Fig. 9—Position of coupling shield used with shields having square cross sections.

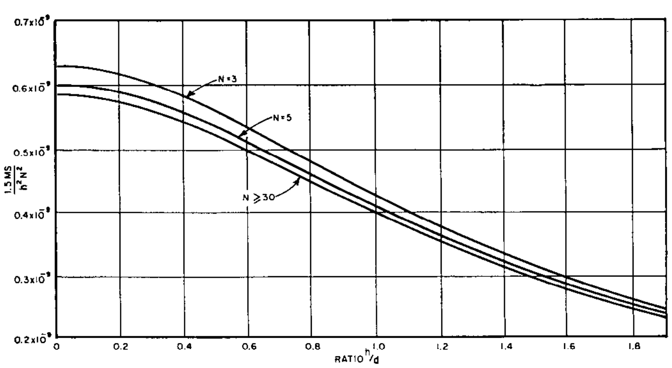

Fig. 10—Mutual inductance between resonators as a function of dimension h.

[10] H. B. Dwight, "Electrical Coils and Conductors," McGraw-Hill Book Co., Inc., New York, N. Y., 1st ed., pp. 257–263; 1945.
[11] *Ibid.*, p. 261.

it can be shown that the ratio $d_0/d = 1/1.32N$. It should be noted that the family of curves of Fig. 10 rapidly converge with increasing N. A value of 1.5 MS/N^2h^2 is obtained from the appropriate curve. Then calculate 1.5 MS/N^2h^2 using the value of h estimated. Unless the original guess was exactly correct, the above two value of 1.5 MS/N^2h^2 will differ. The process is now to pick another value of h until the correct one is determined. This is a rapidly convergent procedure and should generally require about three attempts.

IV. Illustrative Example

In order to complete the design information of a 3-resonator filter, an example will be given. The purpose of the example is to emphasize the fact that the development of the type of filter is a straightforward continuation of regular filter practice in the VHF and UHF domain. It is a missing link between existing low-frequency filter practice, extending from 0 to somewhere above 20 Mc and microwave filters starting in the hundred megacycle region.

A. Specifications

The required filter is of the bandpass variety, centered at 54 Mc with a 3-db bandwidth of 3 Mc. 40-db attenuation is required at 46 and 62 Mc. Insertion loss of the filter must not exceed 1 db. The impedance must be low ($\approx 50\Omega$), and the input and output must have a dc return path. Reflective attenuation in the pass band has to be greater than 13 db or the VSWR has to be no greater than 1.5 (see Fig. 13).

Percentage bandwidth = 3/54 = 5.5 per cent.

B. Solution:

1) Fig. 11 shows the number of resonators necessary for a Butterworth response. Entering with the following information:
   a) Left side—

$$\frac{BW_{40\text{ db}}}{BW_{3\text{ db}}} = \frac{16}{3} = 5.33$$

   b) Right side—absolute value of rejection = 40 db

A straight line connecting these points indicates that necessary number of resonators is 2.8. Therefore 3 resonators will be used. This network will be defined by a third-order polynomial (3-pole network with no zeros).

2) Necessary value of unloaded Q.
   a) The value of $q_{\min}$ for a Butterworth response is 2.0.[12] Then $Q_{\min} = 2.0 \times 54/3 = 36$.
   The rigid necessity to satisfy the requirement of 1-db insertion loss in the pass band at mid-frequency yields the following design relation: L_{DB} times correction factor = 1 db ($= L_{db} \cdot K$).

[12] "Reference Data for Radio Engineers," *op. cit.*, p. 194.

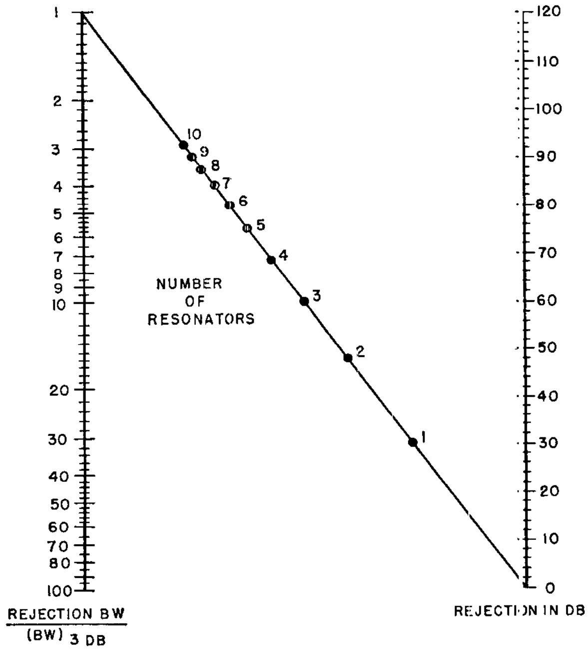

Fig. 11—Number of resonators necessary for a Butterworth response.

   From Fig. 6, the correction factor is seen to be 1.09. $L_{DB} = 1/1.09 = 0.914$ db.
   From Fig. 4, $U = 9.5$ for $L_{db} = 0.914$ db
   $Q_{unl} = U\, Q_{\min} = 9.5 \times 36 = 342$.
   b) Fig. 12 shows an alternate method[13] of finding the unloaded Q. The only limitation upon this nomogram is that the insertion loss per resonator not exceed 3 db. Thus, for a filter whose total insertion loss must not be greater than 1 db, it is obvious that this nomogram is applicable. For this nomogram, Q_{loaded} is defined as $Q_{\text{loaded}} = f_0/BW_{3\text{db}}$. For this example, the insertion loss can be evaluated as follows:

   With a straight edge connect 1 db on the right scale and the point corresponding to 3 resonators. This will intersect 0.053 on the left scale. Then

$$\frac{Q_{\text{loaded}}}{Q_{\text{unloaded}}} = 0.053$$

$$Q_{\text{unloaded}} = \frac{54}{3 \times 0.053} = 340.$$

   It is seen that this value is very close to that value calculated in part "a."

3) Design of Resonator Dimensions.
   a) To insure that the insertion loss remains below 1 db, the helical resonator will be designed for a Q of 500. From (13)–(19),

[13] R. D. Baars, "Nomograms for microwave filter design," *Microwave J.*, vol. 3, pp. 59–62; August, 1960.

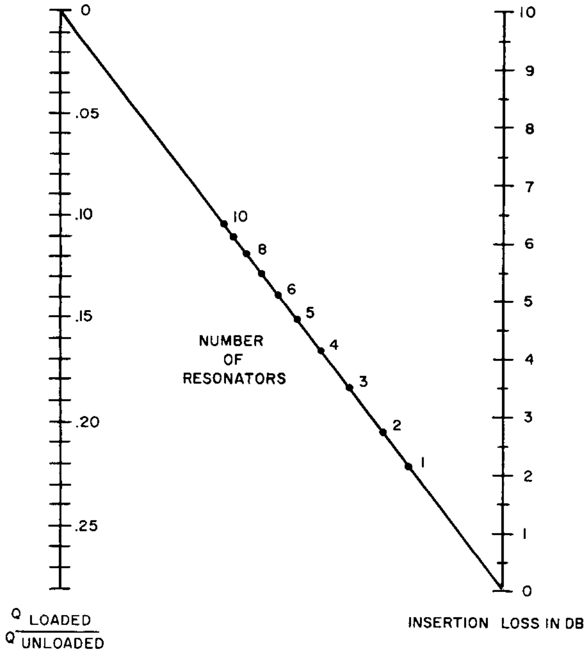

Fig. 12—Insertion loss of a Butterworth filter.

$$S = \frac{Q}{60\sqrt{f_0}} = \frac{500}{60\sqrt{54}} = 1.13 \text{ inches.}$$

Available space allows the use of a standard dimension can whose width is 1.3 inches. For this value of S

$$Q = 60 \times 1.3\sqrt{f_0} = 573.$$

The necessary design quantities are obtained from (14)–(19)

$$N = \frac{1600}{f_0 S} = \frac{1600}{54 \times 1.3} = 22.8 \text{ turns}$$

$$n = \frac{1600}{S^2 f_0} = \frac{1600}{(1.3)^2 \times 54} = 17.6 \text{ turns/inch}$$

$$Z_0 = \frac{81500}{f_0 S} = \frac{81500}{54 \times 1.3} = 1160\Omega$$

$$d = 0.66S = 0.66 \times 1.3 = 0.858 \text{ inches}$$

$$b = S = 1.3 \text{ inches}$$

$$H = 1.6S = 1.6 \times 1.3 = 2.08 \text{ inches.}$$

For $d_0/\tau = 0.5$

$$d_0 = \frac{1}{2n} = \frac{1}{2 \times 17.6} = 0.0284 \text{ inches.}$$

This corresponds to #21 copper wire.

With an available Q of 575, the actual insertion loss of the filter will be lower.

b) An alternate method of determining the dimensions of the resonator makes use of the nomogram of Fig. 2. Connect the center frequency, 54 Mc, on the f_0 scale, with the dimension $S = 1.3$ inches on the right-hand S scale. The line is extended as shown to the Q and n scales. At this point, the straight edge is turned horizontally to the wire size scale. The following can be read:

$$H = 2.1 \text{ inches}$$
$$Q = 580$$
$$n = 17.5 \text{ turns/inch}$$
$$\text{wire} = \#21.$$

Next, connect f_0 with the left-hand S scale, extend the line and read,

$$b = S = 1.3 \text{ inches}$$
$$d = 0.84 \text{ inch}$$
$$Z_0 = 1150\ \Omega$$
$$N = 22 \text{ turns.}$$

These results compare very favorably with the calculated dimensions. The nomogram can be used for a rough design, as a check, or as a method of determining if the resonator to be designed is physically realizable.

4) Dimensions of Coupling Shield.

The dimensions of two coupling shields will now be calculated. From Fig. 10, it is seen that the only dimension unknown is h.

The normalized coefficients of coupling for a 3-resonator equal-resistive terminated filter are

$$k_{12} = k_{23} = 0.707.$$

From (1) the inductance of each coil is

$$L = 0.025(17.6)^2(0.858)^2\left[1 - \left(\frac{0.858}{1.2 \times 1.3}\right)^2\right]$$
$$\times 1.3 = 7.4 \times 0.697 = 5.15\mu h.$$

Because the coefficients of coupling are equal

$M_{12} = M_{23}$ and from (30),

$$M_{12} = M_{23} = 0.707 \times \frac{5.15}{2} \times \frac{3}{54} = 0.101\mu h.$$

From Fig. 10, it is necessary to calculate the quantities represented by the ordinate and abscissa

$$\frac{1.5MS}{N^2h^2} = \frac{1.5 \times 0.101 \times 10^{-6} \times 1.3}{(22.8)^2 h^2} = \frac{0.379 \times 10^{-9}}{h^2} \quad \text{(a)}$$

$$\frac{h}{d} = \frac{h}{0.858} = 1.165\, h. \quad \text{(b)}$$

The final step is, by successive approximations, find a value of h such that (a) and (b) are satisfied on Fig. 10. A value of $h = 1.04$ inches is found to be the correct dimension. Again, referring to Fig. 9, the length of the shield is found to be

$$1.3S - h = (1.3 \times 1.3) - 1.04$$
$$= 1.69 - 1.04 = 0.65 \text{ inches.}$$

Insertion loss $= 1.09 \times L_{DB}$,
where $U = 575/37 = 15.9$
From Fig. 4, $L_{DB} = 0.6$ db.
Thus insertion loss $= 1.09 \times 0.6 = 0.654$ db.

The dimensions of the can are found as follows:

Length $= 3S = 3 \times 1.3 = 3.9$ inches
Width $= S = 1.3$ inches
Height $= 1.6S = 2.08$ inches.

5) The final problem in the filter design is to satisfy the requirement for impedance. This is measured as echo attenuation (return loss), VSWR or reflection coefficient and determines the amount of mismatch between the filter and the terminating impedances. Fig. 13 shows the curve of return loss vs VSWR, where σ, the standing wave ratio, is a function of the reflection coefficient ρ:

$$\sigma = \frac{1 + \rho}{1 - \rho}. \tag{31}$$

At these frequencies, the measurement of return losses is easier to accomplish by means of the bridge method, except for the fact that the voltage source, voltmeter, and bridge itself require resistors of very close tolerance.[14] The desired impedance characteristics can be obtained by adjusting the input- and output-coupling loops. The tuning of the filter can be accomplished by use of a sweep generator, and the response may be measured by the conventional generator-voltmeter method. Fig. 14 shows the filter built to the above specifications and the measured response.

One final remark about the construction of this type of filter. It has been found[15] that a dielectric inside the helix has only a second-order effect while a dielectric outside the helix has a first-order effect when the parameter $(\pi nd)(\pi d/\lambda)$ is considerably less than one. When $(\pi nd)(\pi d/\lambda)$ is greater than 1, Z_0 depends only upon the ratio $(2\lambda/d)$ and introducing dielectric material inside or outside of the helix has equal first-order effect. This fundamental property of the helix can be used for tuning purposes, or at least adjustment to a proper center frequency. Fig. 15 shows the possible construction of a 3-resonator tunable filter. This particular filter was designed for a center frequency from 400 to 450 Mc, while maintaining a constant bandwidth of 10 Mc. In this case three synchronously moving plungers of low-loss dielectric material were used to change the dielectric constant of the medium inside of the helix. The same effect could have been obtained if a cylinder outside was used, instead of a plunger inside the helix.

In Sichak,[15] an example is given of a resonator which could be tuned from 50 to 3500 Mc (a range of 70/1), but no indication of the Q variation was given. It is also indicated, that with proper design, a tuning range of 100/1 is possible.

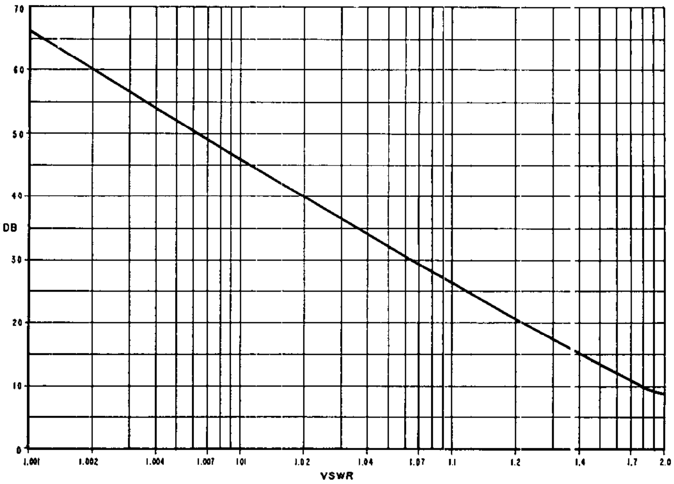

Fig. 13—Return loss vs VSWR.

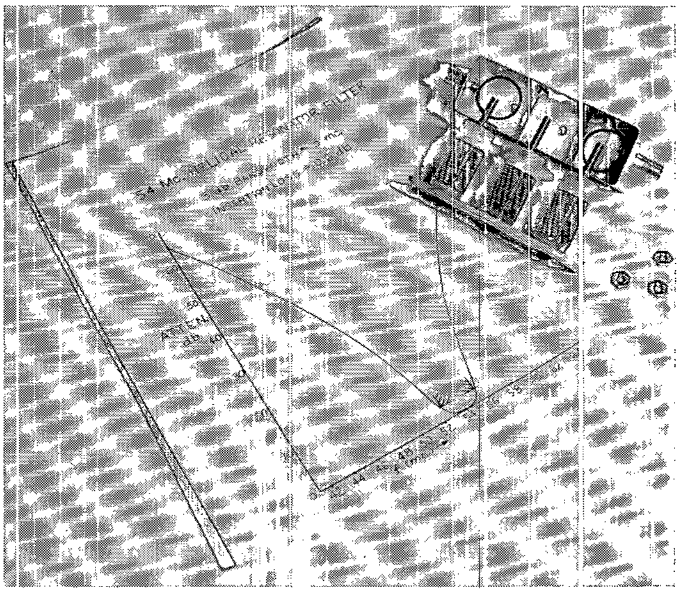

Fig. 14—Helical resonator filter, 54 Mc.

[14] R. E. Lafferty, "Measuring return loss accurately," *Electronic Ind.*, vol. 19, pp. 70–74; October, 1960.
[15] W. Sichak, "Coaxial line with helical inner conductor," *Elec. Commun.*, vol. 32, pp. 62–67; March, 1955.

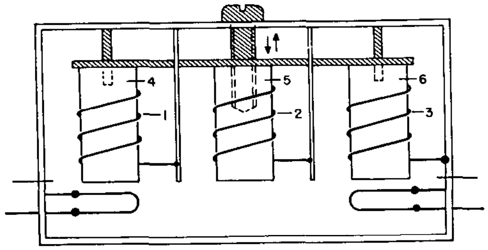

Fig. 15—Tunable filter with helical resonator.

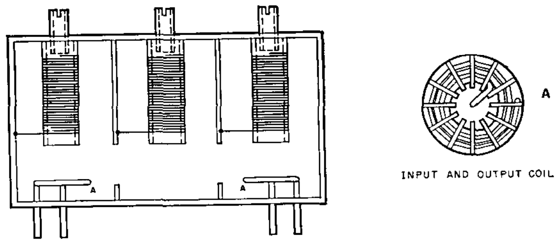

Fig. 16—Filter with helical resonators for printed circuits.

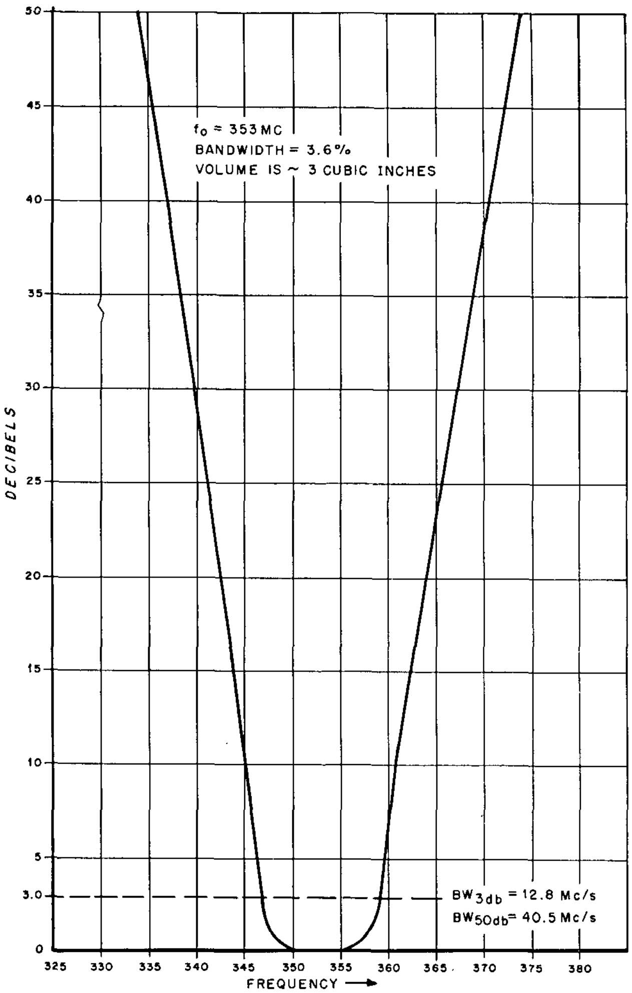

Fig. 17—Response curve of a 5-resonator filter.

Fig 16 shows the construction of a helical resonator filter which has a high input and output impedance while still maintaining a dc return path for the input and output circuit. The assembly of the coupling coils enable the filter to be very well suited for printed circuit applications.

Fig. 17 shows the measured response curve of a 5-resonator filter. The total volume of this filter is only three cubic inches and the equivalent Q of each resonator is approximately 700. The small size and low insertion loss of this filter emphasize the advantage of using helical resonators as filter elements.

Finally, Fig. 18 shows the construction and response curve of a 7-resonator filter whose center frequency is 30 Mc. The coupling between resonators is provided by metal shields, and the filter is tuned by a brass screw at the top of each coil. Fig. 19 shows the same filter completely assembled. The brass screws are locked by the nuts to insure that the center frequency does not shift.

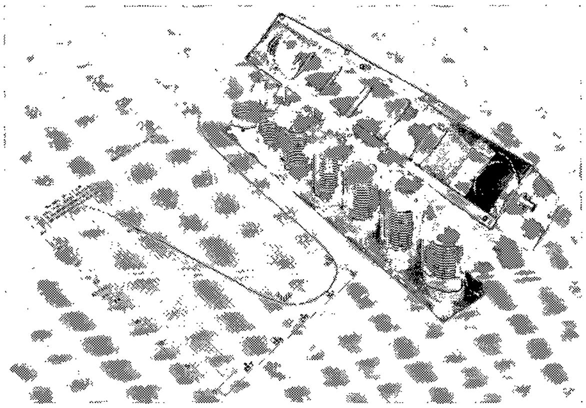

Fig. 18—30-Mc filter with cover off.

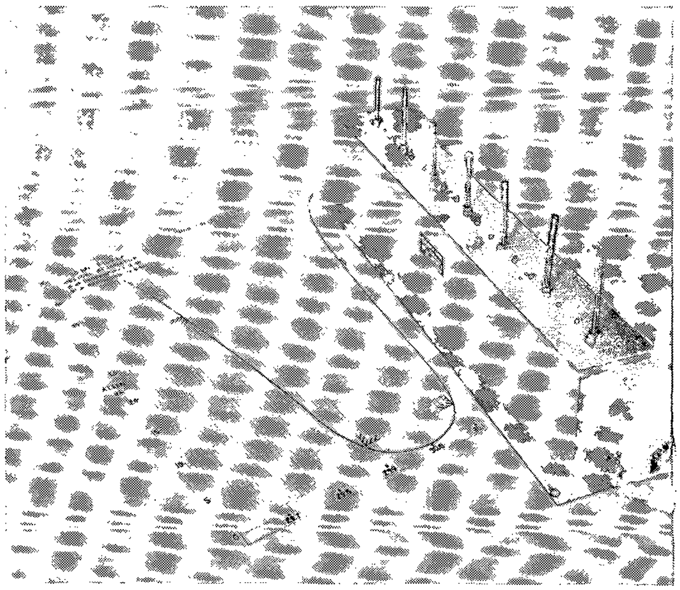

Fig. 19—30-Mc filter with cover on.

V. Conclusion

Filters with helical components simplify the filtering problem at frequencies where previously only lumped constant elements and coaxial resonators were employed. Use of these filters solves the problems of excessive pass-band insertion loss, when lumped-constant elements are used, and of unreasonably large size and weight, when coaxial resonators are used.

At Westinghouse Electronics Division, the helical resonator has proved to be a usable addition to conventional components, such as solenoids, toroids, piezoelectric crystals, coaxial resonators, and waveguides. Consistant experimentation has proved that practical high-quality passive filtering with an unlimited number of resonators in the composite network can be accomplished from approximately 20 Mc to 2 Gc. At high frequencies the size of the helical resonator filter is so small that no available passive filter is able to compete with it. This type of filter is a standard feature in our VHF and UHF receivers and transmitters and is a part of the radar with antijamming features.

The military requires that electronic equipment pass certain temperature, shock, and vibration tests. After the mechanical assembly of the input and output coupling loops was slightly modified, the helical resonator filters successfully met the military requirements.

Acknowledgment

The authors wish to thank M. Savetman for carrying out the original experiments and collecting the necessary data, S. Russell for his contributions which helped reduce the application problems to an engineering level, and T. Oray for his contributions to the coupling problem.

The Split-Feedback Push-Pull Magnetic Amplifier*

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Summary—**This paper describes a high-gain phase-sensitive magnetic amplifier which simultaneously possesses the high-gain property of the self-saturating (100 per cent positive feedback) amplifiers and the limited circulating current property of the low-gain zero feedback amplifiers. These features are obtained by using feedback windings to obtain the self-saturating large-gain property. However, the feedback windings are arranged so that the feedback is positive only when load current flows and is zero when circulating currents flow. The circuit is analyzed and its various modes of operation are described. The theory is verified with numerous experimental results.**

Introduction

The split-feedback push-pull magnetic amplifier is a high-gain, phase-sensitive amplifier in which the magnitude of the circulating currents is considerably less than that in others of equal gain and power rating. Circulating push-pull currents are an inherent property of phase-sensitive amplifiers for the following reason. The amplifier circuit consists of two meshes with the load common to each mesh [Fig. 1(a)]. The nature of the unconstrained magnetic amplifier is such that current flows in any mesh during a portion of the cycle only. It is convenient to divide the half cycle of excitation frequency into the following regions [see Fig. 1(b)]:

1) From 0 to A_1 when neither mesh conducts and the two mesh currents $i_1 = i_2 = 0$[1].
2) From A_1 to A_2 when only one mesh conducts; $i_1 = v/R_t$, $i_2 = 0$ where v is the applied EMF per mesh and $R_t = R + r$ is the sum of the load resistance R common to both meshes and r is the other resistance (winding, rectifier forward resistance, etc.) in the mesh.
3) From A_2 to π when both meshes (assumed identical) conduct so $i_1 = v/r = i_2$.

Load current $i = i_1 - i_2$ flows only in region 2 from A_1 to A_2, while from A_2 to π, very large currents, depending on the ratio of R_t to r, flow around the outside loop. The magnetic amplifier must be designed on the basis of the maximum dissipation in its windings. The circulating currents are therefore often the limiting factor.

For the purpose of discussing circulating currents, it is appropriate to distinguish between two amplifier categories. One of these may be called the "current amplifier" because the output current is (over the amplifying range) some constant times the input current, independent of the load or the excitation voltage. There is no circulating current problem here. The mesh currents are held within bounds and although the circulating

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