Phys 371 : Lecture 8 : 2/17/2016

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**Review** : Postulates of SR :

① The speed of light is constant in all reference frames

② The laws of physics are the same in all inertial frames.

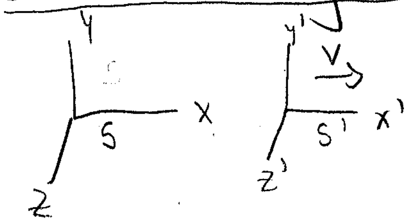
**3 Fundamental Consequences**

① Loss of Simultaneity

☆ ② Time Dilation :

☆ ③ Length Contraction

**Described by Lorentz Transformations :**




$$\left.\begin{aligned} x' &= \gamma(x - vt) \\ y' &= y \\ z' &= z \\ t' &= \gamma(t - vx/c^2) \end{aligned}\right\}$$ Consider Inverse Transformations

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \geq 1$$

**Time Dilation**

Two Events happen at same position in S

$X = 0$ for simplicity, with time interval $\tau_0$

$t_1 = 0$ $t_2 = \tau_0$

In S', from LT : $\boxed{\tau' = \gamma \tau_0}$

Time interval is longer, clock runs "slow".

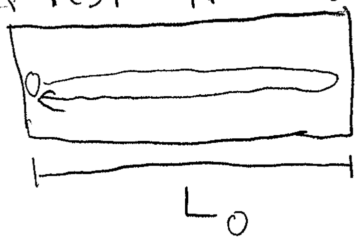
Looking from other frame not a contradiction. Result relies on clocks being at rest in a given frame.

– More Intuitive derivation on HW. "Light clock"

# Length Contraction

Measure as round trip of light pulse on train

In rest frame of train

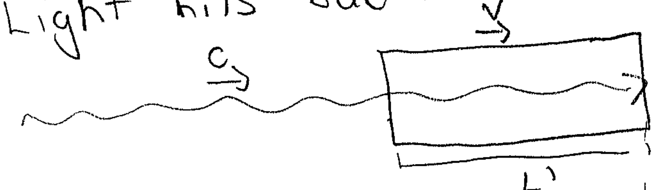


$$\tau_0 = \frac{2L_0}{c} \Rightarrow L_0 = \frac{c\tau_0}{2}$$

In Ground Frame

① Light Emitted

② Light hits back @ $\Delta t_1$

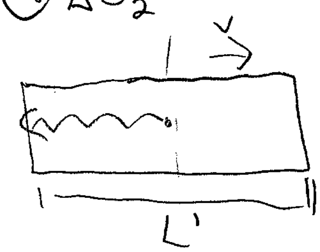


$$\Delta t_1 = \frac{L'}{c} + \frac{\Delta t_1 v}{c}$$

$$\Delta t_1 (1 - v/c) = L'/c$$

$$\Delta t_1 = \frac{L'}{c - v}$$

③ Light hits front @ $\Delta t_2$



$$\Delta t_2 = \frac{L'}{c} - \frac{\Delta t_2 v}{c}$$

$$\Delta t_2 = \frac{L'}{c+v}$$

Total Round Trip: $\tau_G = \Delta t_1 + \Delta t_2 = \frac{L'}{c-v} + \frac{L'}{c+v}$

$$\tau_G = \frac{2L'c}{c^2 - v^2} = \frac{2L'}{c}\gamma^2$$

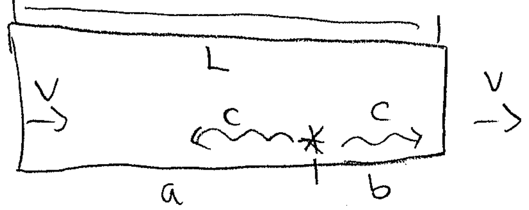
But the Ground observer sees the train clock running slower

$$\tau_G = \gamma\tau_0 \Rightarrow \frac{2L'}{\cancel{c}}\gamma^{\cancel{2}} = \cancel{\gamma}\left(\frac{2L_0}{\cancel{c}}\right)$$

$\boxed{L' = L_0/\gamma}$ ← Length Shortens as seen from moving frame.

Phys 371 Mental Aerobics 2/17/2016

A train with proper length $L$ moves to at speed $v$. Where should the light source be located on the train, so than an observer *on the ground* sees the photons hit the front and back of the train at the same time?




$L = a + b$ (Train Frame)

$a' = a/\gamma$, $b' = b/\gamma$ (Length Contraction)

Ground Frame | $t_b' = \frac{b/\gamma}{c} + \frac{t_b V}{c}$

$t_b' = \frac{b}{\gamma c} \frac{1}{(1 - V/c)} = \frac{b}{\gamma(c-V)}$

$t_a = \frac{a/\gamma}{c} - \frac{t_a V}{c} \Rightarrow t_a = \frac{a}{\gamma c} \frac{1}{(1+V/c)} = \frac{a}{\gamma(c+V)}$

Same Arrival Time in Ground Frame | $t_a' = t_b$ $\quad \frac{a}{c+V} = \frac{b}{c-V}$

Use: $L = a + b \Rightarrow \frac{L-b}{c+V} = \frac{b}{c-V}$ $\quad Lc - LV - bc + \cancel{bV} = cb + \cancel{bV}$

$\boxed{b = \frac{L(c-v)}{2c}}$

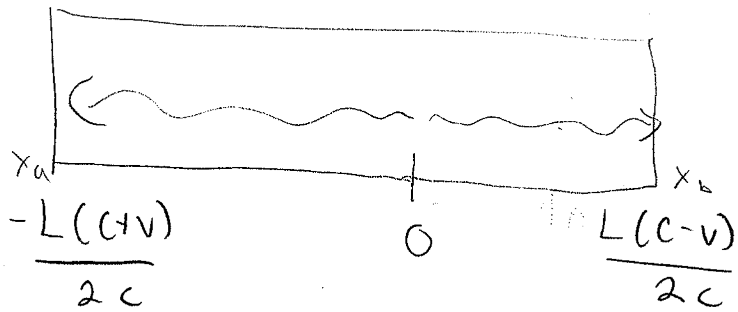
$a = L - \frac{L(c-v)}{2c} = \frac{2cL - cL + Lv}{2c} = \boxed{\frac{L(c+v)}{2c} = a}$

**Feedback:** Today's lecture is

Too fast about right too slow

Other comments: ______________________________

**Exercise:** Use results to confirm L-contraction using Lorentz transforms




$t'_a = t'_b$

$$-\left(\frac{L(c+v)}{2c}\right) = \gamma(x'_a - v t'_a)$$

$$\frac{L(c-v)}{2c} = \gamma(x'_b - v t'_b)$$

**Diff:** $$\frac{L(c-v)}{2c} + \frac{L(c+v)}{2c} = \gamma(\overbrace{x'_b - x'_a}^{L'})$$

$$L \frac{2\cancel{c}}{2\cancel{c}} = \gamma L' \Rightarrow \boxed{L' = L/\gamma}$$

**Real Life Examples:**

**Muons:** Created in upper atmosphere, a live w/ lifetime 2 $\mu s$, with $v = (1 - 2\times10^{-5})c$. If they are created 50 km up, do they hit the ground.

**Newton Would Have Said No:** $d = vT \approx 600\ m < 50\ km$

**SR from Observer's Perspective:** $d = v(\gamma T) = \frac{600\,m}{\sqrt{1-(1-2\times10^{-5})^2}} \approx \frac{600\ m}{160} \approx 100\,km$

Time Dilation → Yes!

**SR from Muon's Perspective:** $d/\gamma = vT \Rightarrow T_E = \frac{d}{\gamma v} = \frac{50\ km}{160\ (3\times10^8)}$
$\approx 1\ \mu s$

Length Contraction Yes!

## World's Best Atomic Clock:

Measure a tick frequency

$$\nu_0 = \frac{1}{\tau_0} \qquad \nu' = \frac{1}{\tau'} = \frac{1}{\gamma \tau_0} = \frac{\nu_0}{\gamma}$$

rest frame ↗ moving frame ↗

How stable does a slow clock need to be?

Measure a relative shift in frequency

$$\frac{\Delta\nu}{\nu_0} = \frac{\nu' - \nu_0}{\nu_0} = \frac{\nu_0 (1 - 1/\gamma)}{\nu_0} = 1 - \sqrt{1 - v^2/c^2}$$

$$\approx 1 - (1 - \tfrac{1}{2} v^2/c^2)$$

$$\boxed{\frac{\Delta\nu}{\nu_0} \approx \frac{1}{2}\frac{v^2}{c^2}}$$

for 1 m/s $\quad \frac{\Delta\nu}{\nu_0} \approx 0.5 \times \frac{1}{9 \times 10^{16}} \approx 5 \times 10^{-18}$

Just at the limit of world's most precise atomic clocks. (Lose 1 sec is 4 billion years)

Chou et al (Science 2010)

measure $v = 5$ m/s $\quad \frac{\Delta\nu}{\nu} \approx 1.4 \times 10^{-16}$

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Phys 379: Lecture 9

① Real Life Examples from last lecture (Muons + clocks)
② Velocity Addition
③ Doppler Effect

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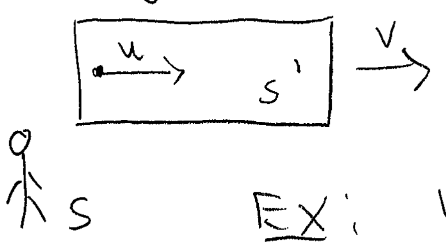
## Velocity Addition




What speed does the ball have relative to the ground?

Ex: $u = \frac{1}{2}c$, $v = \frac{3}{4}c$

Newton would say: $v_g = u + v = \frac{5}{4}c > c$ ✗

Derive Relativistic Result from Lorentz Transformations

$$\Delta x = \gamma(\Delta x' + v\Delta t')$$
$$\Delta t = \gamma(\Delta t' + v\Delta x'/c^2)$$

$u = \frac{\Delta x'}{\Delta t'}$ and $v_g = \frac{\Delta x}{\Delta t}$

Then: $$v_g = \frac{\Delta x' + v\Delta t'}{\Delta t' + v\Delta x'/c^2} = \frac{\overbrace{\Delta x'/\Delta t'}^{u} + v}{1 + v/c^2\,\underbrace{\Delta x'/\Delta t'}_{u}}$$

$$\boxed{v_g = \frac{u+v}{1+uv/c^2}}$$

Important Properties:

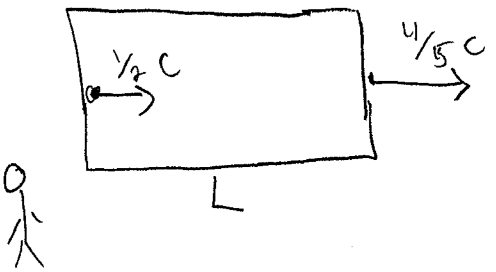
✓ ① Symmetric ($u \Leftrightarrow v$)

✓ ② $uv \ll c^2$, $v_g \approx u + v$ (Galilean velocity Transform)

✓ ③ If $u, v = c$, $v_g = c$ (speed of light same in all frames)

Ex from Above: $$v_g = \frac{(\frac{1}{2} + \frac{3}{4})c}{1 + (\frac{1}{2})(\frac{3}{4})c^2/c^2} = \frac{5/4\,c}{11/8} = \frac{10}{11}c$$

A ball is launched with speed $u = \frac{1}{2}c$ from the back of a train car with proper length $L$. If the train travels with speed $v = \frac{4}{5}c$, after how much time, according to an observer on the ground, does it take for the ball hit the front of the train car? How far does it travel according to this observer?




Velocity of Ball relative to observer:

$$V_b = \frac{\frac{1}{2}c + \frac{4}{5}c}{1 + (\frac{1}{2})(\frac{4}{5})\frac{c^2}{c^2}} = \frac{13/\cancel{10}^{2}}{7/\cancel{5}}\, c = \frac{13}{14} c$$

The ball travels the contracted length of the train plus the extra distance it travels.

$$\gamma = \frac{1}{\sqrt{1 - 16/25}} = \frac{5}{\sqrt{9}} = \frac{5}{3}$$

$$t_b = \frac{L/\gamma}{V_b} + \frac{t_b V}{V_b} \Rightarrow t_b = \frac{\frac{L}{\gamma}}{V_b - V} = L\left(\frac{3}{5}\right)\frac{1}{\left(\frac{13}{14} - \frac{4}{5}\right)c}$$

$$= L\left(\frac{3}{5}\right)\frac{70}{9} = \boxed{\frac{14}{3} L/c}$$

The distance is simply

$$\Delta x_b = V_b t_b = \left(\frac{13}{\cancel{14}} \cancel{c}\right)\left(\frac{\cancel{14}}{3} \frac{L}{\cancel{c}}\right) = \boxed{\frac{13}{3} L}$$

Same Procedure Gives <u>Transverse Velocities</u>

$$v_y = \frac{\Delta y}{\Delta t} = \frac{\Delta y'}{\gamma(\Delta t' + v\,\Delta x'/c^2)}$$

$$\boxed{v_y = \frac{u_y'}{\gamma(1 + v u_x/c^2)}}$$

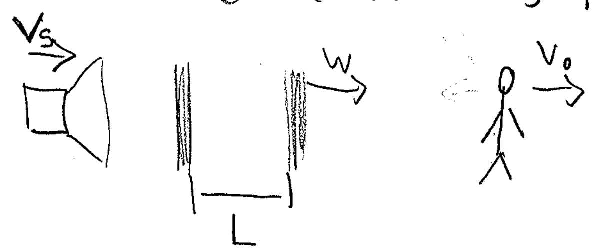
<u>Limerick</u>:

For a bullet, a train, and a gun
Adding the speeds can be fun
Take a trip down the path
Paved with Einstein's new math,
Where a half plus a half isn't one.

<u>Exercise</u>: <u>Ball on Train</u>

<u>Doppler Effect</u>:

In sound (Matter):




Period
$$T_o = \frac{1}{\nu_o}$$
Frequency

Wavelength
$$\lambda_o = \frac{w}{\nu_o}$$
speed

<u>Moving Source</u>: Distance between pulses compressed (wavelength modified)

$$\lambda_D = \lambda_o - vT = w/\nu_o - v/\nu_o = \frac{1}{\nu_o}(w - v_s)$$

$$\boxed{\nu_D = \nu_o \frac{1}{1 - v_s/w}} \approx \nu_o(1 + v_s/w)$$

<u>Moving Observer</u>: Relative velocity of wave changes

$$\nu_D = \frac{(w - v_o)}{\lambda_o} = \nu_o(1 - v_o/w)$$

Phys 371 ; Lecture 10 : 2/22/2016

① Doppler Effect (Classical + Relativistic)

② SR Paradoxes

## Classical Doppler Effect

$$\nu_D = \nu_0 \left( \frac{1 - V_o/w}{1 + V_s/w} \right)$$

## Relativistic Doppler Effect

Consider flashing light source + observer. with relative velocity $v$


Frequency Modified Due to

① Time Dilation

$$\tau = \gamma \tau_0$$

② Wavelength Compression

$$\lambda_D = c\tau - v\tau = (c - v)\tau$$

$$\nu_D = \frac{c}{\lambda_D} = \frac{c}{c-v}\frac{1}{\gamma\tau_0} = \frac{1}{1-v/c}\frac{\sqrt{1-v^2/c^2}}{\tau_0} = \frac{\sqrt{(1+v/c)(1-v/c)}}{(1-v/c)}\frac{1}{\tau_0}$$

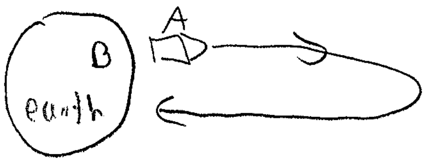
$$\boxed{\nu_D = \nu_0 \sqrt{\frac{(1+v/c)}{(1-v/c)}}}$$

Checks ; ① If $v > 0$, $\nu_D/\nu_0 > 1$. Light is "blueshifted"

② $v/c \ll 1 \Rightarrow \nu_D \approx \nu_0 (1 + \tfrac{1}{2} v/c)(1 + \tfrac{1}{2} v/c) \approx \nu_0 (1 + v/c)$

Agrees w/ Classical result

## SR Paradoxes: Who is Younger?



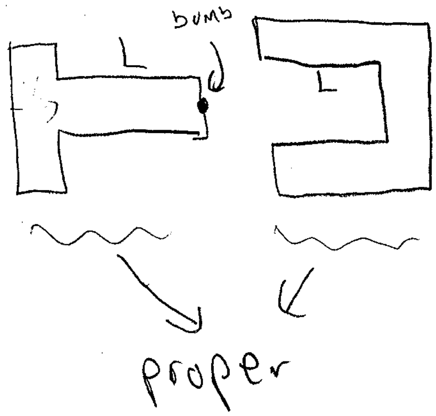

A sees B's clock run slow, + vice-versa.

But the situation is not Symmetric. A must accelerate to turn around.

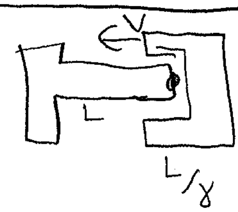
$\boxed{\text{A is younger}}$

While A is accelerating, sees A's clock runs extra slow from GR redshift time dilation.

## T and u blocks: Travel toward each other with relative velocity V.

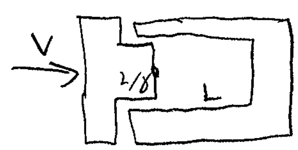



In T-frame:




u is contracted
Bomb explodes!

In u-frame




Will it be stopped?

But signal has to propagate to front to stop bomb. At best info travels at c

Bomb will explode if

$$\frac{L - L/\gamma}{V} < \frac{L}{c}$$

$\frac{L}{c}$ ← time it takes stopping signal to propagate down block

$\frac{L - L/\gamma}{V}$ ← time it takes bomb to go remaining distance

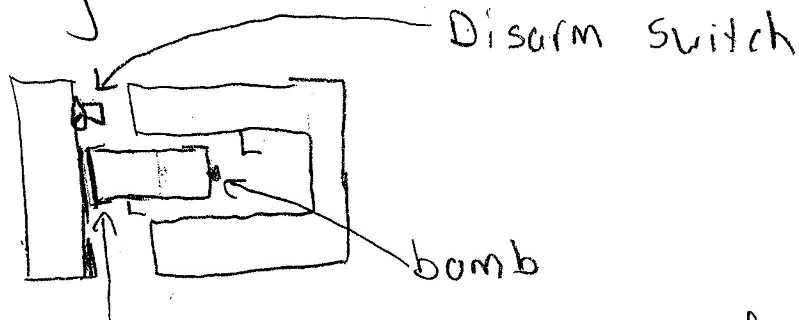
$$1 - \frac{1}{\gamma} < \beta \Rightarrow 1 - \sqrt{1-\beta^2} < \beta \Rightarrow 1 - \beta < \frac{\sqrt{1-\beta^2}}{\sqrt{(1-\beta)(1+\beta)}}$$

$$\Rightarrow \sqrt{1-\beta} < \sqrt{1+\beta} \quad \checkmark$$

True for any $\beta$!

The way we have formulated the problem implies the "T-block" will stretch until the bomb makes contact.

If you find this hard to accept, consider the following variation of the T-block.




This port can slide forward if the back stops.

Now the condition of the bomb exploding depends on whether or not the disarm switch hits first. But in the "U-block" frame, our arguement from above still holds. The signal from the disarm switch can travel to the bomb at a max velocity of c. It is still true that this is not fast enough to catch up with the bomb.

So whether it is delayed disarm signals or stretching molecular bonds, the resolution to the paradox is the same. All observers agree that the bomb explodes.