Phys 371 | Lecture 4 | 2/5/2016

① "Pre-Modern Physics"
- Newton's Laws (1687) – Kinematics + dynamics
- Maxwell's Equations (1861) – Electromagnetism + light

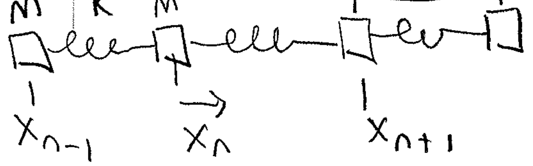
② "Modern Physics"
- Einstein's Relativity
  - ☆ – Special Relativity (1905) → Fix Newton's equations for $V \gtrsim c$. Define space-time
  - – General Relativity (1915) → Gravity ≡ curvature of space-time
- ☆ – Quantum Mechanics (1900 – 1930s) → Description of the very small

③ "Contemporary Physics"
- ☆ – Quantum Information (1980s – today) → Using QM to perform tasks impossible for classical Devices.

Review #1: Waves in Mechanial Systems

From Newtonian Physics (F = ma)

Example: Compressive waves in a mass/spring system



$ma = F$

$$m\ddot{x}_n = k(x_{n+1} - x_n) - k(x_n - x_{n-1})$$

Continuum Limit $\begin{cases} \Delta x \to 0 \\ m \to 0 \\ k \to \infty \end{cases}$ while $\mu \equiv \dfrac{m}{\Delta x} \Rightarrow$ "density"

$K \equiv k\Delta x \Rightarrow$ "Youngs Modulus"

$X_n(t) \rightarrow f(x,t) \equiv$ displacement of mass at x at time t

$$m \frac{\partial^2 f(x,t)}{\partial t^2} = K \left.\frac{\partial f(x,t)}{\partial x}\right|_+ \Delta x - K \left.\frac{\partial f(x,t)}{\partial x}\right|_- \Delta x$$

$$= K \Delta x^2 \frac{\partial^2 f(x,t)}{\partial x^2}$$

For above, we used: $\frac{\partial f}{\partial x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$

$$\boxed{\frac{\partial^2 f}{\partial t^2} = \frac{K \Delta x^2}{m} \frac{\partial^2 f}{\partial x^2} = \frac{K}{\mu} \frac{\partial^2 f}{\partial x^2} \Rightarrow \text{The wave Equation!}}$$

Recall soln is $f(x,t) = f(x \pm c_w t)$ eg $\rightarrow \sin(x - c_w t)$

(Exercise ?) where $c_w = \sqrt{\frac{K}{\mu}} \Rightarrow$ Speed

Wave speed a property of the medium!

In'l general:

$$c_w = \sqrt{\frac{\text{Stiffness}}{\text{Inertia}}}$$

Solids: eg steel = $\kappa = 160$ GPa, $\rho = 7{,}700$ kg/m^3 $\Rightarrow c_w = 6{,}000$ m/s $\approx 13{,}500$ mph

Gas: $c_w = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma(nRT/V)}{nM/V}} = \sqrt{\frac{\gamma R T}{M}}$

$$\boxed{c_w \sim 340 \text{ m/s}}$$

Review #2: EM wave (light) propagation

Maxwell's Eqns In Free Space

1. $\vec{\nabla} \cdot \vec{E} = 0$
2. $\vec{\nabla} \cdot \vec{B} = 0$
3. $\vec{\nabla} \times \vec{E} = -\partial B / \partial t$
4. $\vec{\nabla} \times \vec{B} = \mu_0 \varepsilon_0 \dfrac{\partial \vec{E}}{\partial t}$

Gradient Operator

$$\vec{\nabla} = \frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}$$

Recall "Curl of Curl" (3) $\rightarrow$ $\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \vec{\nabla}(\overset{0\ (1)}{\cancel{\vec{\nabla} \cdot \vec{E}}}) - \nabla^2 E$

$$-\nabla^2 E = -\frac{\partial}{\partial t} \vec{\nabla} \times \vec{B} = -\frac{\partial}{\partial t}\left(\mu_0 \varepsilon_0 \frac{\partial E}{\partial t}\right)$$

$$\boxed{\frac{\partial^2 E}{\partial t^2} = \frac{1}{\mu_0 \varepsilon_0} \frac{\partial^2 \vec{E}}{\partial x^2}}$$

The Wave Equation!

$$C_w = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8 \text{ m/s}$$

If there is a medium, must be very stiff + very light.

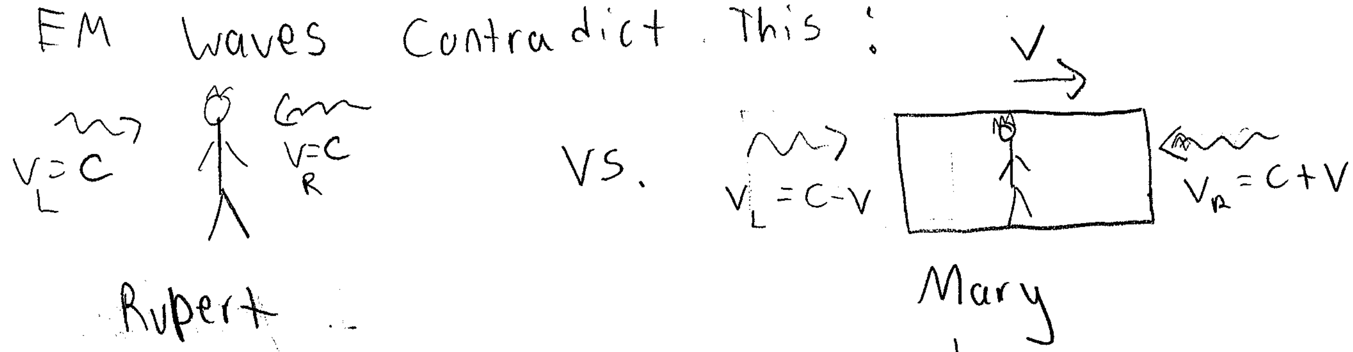
The Problem: Newton's Laws + EM waves are inconsistent

Def: Inertial Frame = A system of coordinates in which all laws of physics take usual form.

Eg: In Newtonian mechanics, Frames traveling w/ constant relative velocity are inertial.

Rail car #1

EM waves Contradict This:



↓ Sees light with two different velocities. Contradicts $C = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$ in vacuum

The Fix:

① Maxwell's equations only true in a preferred frame, $\stackrel{\text{Def}^{\text{n}}}{=}$ "the ether". very stiff medium for EM wave propagation

- Rupert at rest in ether: $V = C$ always
- Mary is moving through ether: Light velocity depends on her motion relative to ether.

② Velocity addition in newtonian mechanics must be wrong.

① ✗ Disproved by experiment (Next week)

② ✓ Einstein's special relativity