HW2 Solution

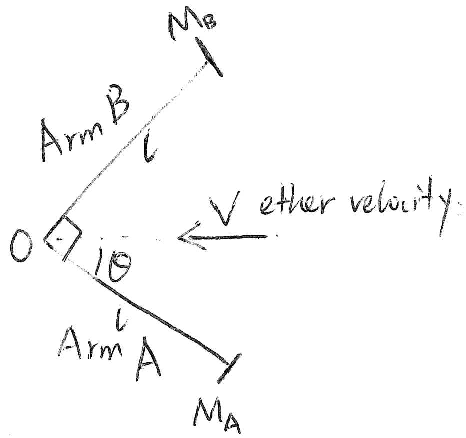
12.2

$$N = \frac{2l}{\lambda}\left(\frac{V}{c}\right)^2$$

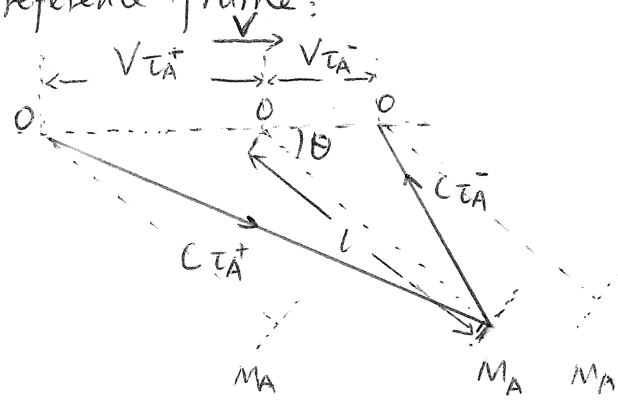
$N = 0.01$ $\quad\lambda = 590\,nm$ $\quad l = 1\,m$

We can get: $\quad V = 4.91\ km/s$.

12.3



Focus on Arm A, in ether's inertial reference frame:



As shown in plot, $\tau_A = \tau_A^+ + \tau_A^-$, $V\tau_A^-$, $C\tau_A^-$ form a triangle with one angle θ. Using Cosine Theorem.

$$(C\tau_A^-)^2 = (V\tau_A^-)^2 + l^2 - 2V\tau_A^- l\cos\theta$$

Similarly, we can get:

$$(C\tau_A^+)^2 = (V\tau_A^+)^2 + l^2 + 2V\tau_A^+ l\cos\theta$$

Then, we can solve for τ_A^+ and τ_A^-, to get:

$$\tau_A = \tau_A^+ + \tau_A^- = \frac{2l}{c^2 - V^2}\sqrt{c^2 - V^2\sin^2\theta}$$

(You can check when $\theta = 0$ and $\theta = 90°$, this gives right answer as shown in textbook.)

Then we can get τ_B from Arm B.

$$\tau_B = \tau_B^+ + \tau_B^- = \frac{2l}{c^2-v^2}\sqrt{c^2-v^2\cos^2\theta}$$

Then time difference $\Delta\tau$:

$$\Delta\tau = \tau_A - \tau_B = \frac{2l}{c^2-v^2}\left(\sqrt{c^2-v^2\sin\theta} - \sqrt{c^2-v^2\cos^2\theta}\right)$$

$$\approx \frac{2lc}{c^2-v^2}\left[\frac{1}{2}\frac{v^2}{c^2}(\cos^2\theta - \sin^2\theta)\right]$$

$$\approx \frac{l}{c}\frac{v^2}{c^2}\cos 2\theta$$

12.6 (a).

$$t_{A\to B} = \frac{l}{c+v}$$

$$t_{B\to A} = \frac{l}{c-v}$$

$$\Delta T = t_{B\to A} - t_{A\to B} = \frac{l}{c-v} - \frac{l}{c+v}$$

$$= \frac{2lv}{c^2-v^2}$$

$$\approx \frac{2lv}{c^2}\left(1+\frac{v^2}{c^2}\right)$$

$$= \frac{2l}{c}\frac{v}{c} + \frac{2l}{c}\left(\frac{v}{c}\right)^3$$

$$\approx \frac{2l}{c}\left(\frac{v}{c}\right) \quad \text{(drop } o\left(\tfrac{v}{c}\right)^3 \text{ term)}$$

(b).

$$\Delta T = \frac{24\times 60\times 60}{10^6}\ s$$

$$l = 5.6\times 6.4\times 10^6\ m$$

$$\Delta T = \frac{2lv}{c^2} \Rightarrow v_{min} = \frac{\Delta T c^2}{2l} \approx 1.08\times 10^{-2}\ m/s$$

$$\approx 1.1\times 10^{-2}\ m/s$$