

1. Solve the below ordinary differential equations, where  $C$  is a positive constant. Include and label any integration constants.

$$(a) \frac{dy}{dx} = Cx \quad (b) \frac{d^2y}{dx^2} = Cy \quad (c) \frac{d^2y}{dx^2} = -Cy \quad (d) \frac{d^2y}{dx^2} = Cx$$

2. Provide the form of  $f(x,t)$  in the below partial differential equations, where  $C$  is a positive constant. You don't have enough information to solve it, but simply show how the function  $f$  must depend upon  $x$  and  $t$ .

$$(a) \frac{\partial^2 f(x,t)}{\partial t^2} = C \frac{\partial^2 f(x,t)}{\partial x^2} \quad (b) \frac{\partial f}{\partial t} = C \frac{\partial^2 f}{\partial x^2}$$

3. Simplify the below expressions into a sum of real and imaginary parts  $a+bi$

$$(a) i^i \quad (b) \tan(i) \quad (c) \log(i) \quad (\text{this is a natural log to base } e, \text{ also known as } \ln)$$

4. Compute the normalization constant  $C$ , and the mean and variance of the following continuous probability distributions of the random variable  $x$ , define over the domain indicated.

$$(a) f(x) = Ce^{-|x|/a} \quad (-\infty < x < \infty) \quad (b) f(x) = C \sin(x/a) \quad (0 < x < a\pi)$$

$$(c) f(x) = C \quad (a < x < b) \quad (d) f(x) = C \delta(x-a) \quad (-\infty < x < \infty)$$

*Note:  $\delta(x)$  is the Dirac delta function*

5. A black box flips two coins, but you cannot observe the coins. Instead, the machine tells you any ONE of three pieces of information given a single trial:

- (i) the results of coin #1 (H or T),
- (ii) the results of coin #2 (H or T), or
- (iii) whether the coins matched or were different.

After a huge number of independent trials, you observe that

- when coin #1 is reported, it is Heads (H) 2/3 of the time,
- when coin #2 is reported, it is also Heads (H) 2/3 of the time, and
- when the coins are compared, they are always different (!)

Write down the matrix of joint probabilities for the four possibilities of outcomes HH, HT, TH, and TT. Interpret your answer, especially if you see any strange entries.