Physics 371 Homework #10 Due: Monday May 2, 2016

1. In this problem we use the following result. If heat is added to a system that has a constant heat capacity C, raising the absolute temperature from T_i to T_f, the change in entropy of the system is

$$\Delta S = \int \frac{dQ}{T} = \int_{T_i}^{T_f} \frac{CdT}{T} = C \ln\left(\frac{T_f}{T_i}\right)$$

Two identical blocks of aluminum, each with C = 300 J/K, start out at absolute temperatures 300 K and 400 K. They are put into thermal contact but are otherwise isolated. Intuitively, we can see that they both end up at 350 K, the temperature exactly halfway in between.

(a) How much heat is transferred in the process?

(b) What is the entropy change for each block?

(c) What is the overall entropy change for the whole system?

2. Why are German words typically longer than English words? Hint: consider the following probability distribution of the usage of the letters in the two languages. Explain your answer IN WORDS in addition to the math:

letter	P(English)	P(German)	letter	P(English)	P(German)	Letter	P(English)	P(German)
a	0.073	0.0651	j	0.002	0.0019	S	0.063	0.0677
b	0.009	0.0257	k	0.003	0.0188	t	0.093	0.0674
c	0.030	0.0284	l	0.035	0.0283	u	0.027	0.0370
d	0.044	0.0541	m	0.025	0.0301	v	0.013	0.0107
e	0.130	0.1673	n	0.078	0.0991	w	0.016	0.0140
f	0.028	0.0204	o	0.074	0.0229	x	0.005	0.0002
g	0.016	0.0365	p	0.027	0.0094	y	0.019	0.0003
h	0.035	0.0406	q	0.003	0.0006	z	0.001	0.0100
i	0.074	0.0781	r	0.077	0.0654			

3. Consider a normal deck of 52 distinct playing cards. A new deck always is prepared in the same order (A♠ 2♠ · · · K♣).
(a) What is the information entropy of the distribution of new decks?
(b) What is the information entropy of a distribution of completely randomized decks?
(c) Computer simulations show that by properly shuffling a new deck 9 times, the deck will become essentially random. About how much information entropy is added to the deck for each shuffle?

4. A logic gate takes two binary inputs A and B and produces two binary outputs A′ and B′. Assuming that each possible input state is equally likely, compute the Shannon entropy $H = -\sum_i p_i log_2 p_i$ of the input and the output, and the change in Shannon entropy for the following cases:

   (a) The gate is given by two NOT operations (NOT operation: 0→1 and 1→0)

   A′ = NOT A
   B′ = NOT B

   (b) The gate is given by

   A′ = A OR B
   B′ = A AND B