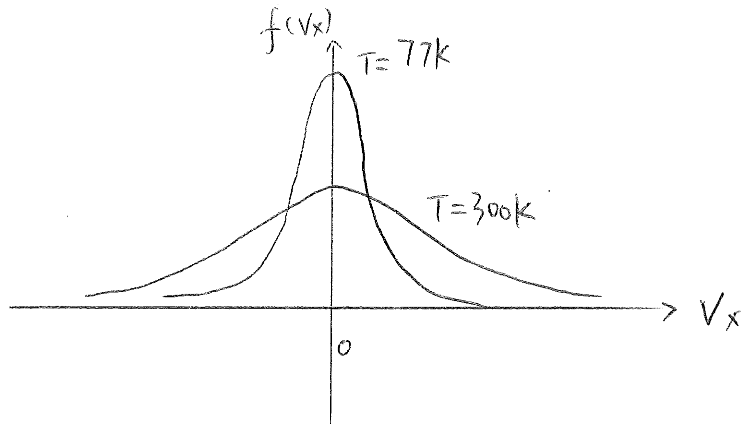
HW9 Solution

Problem 1:

Maxwell-Boltzmann distribution for velocity:

$$f(V_x) = \sqrt{\frac{m}{2\pi kT}}\, e^{-\left(\frac{mV_x^2}{2kT}\right)} \quad \text{(velocity along one direction)}$$



In order to calculate root-mean-square value of velocity, we need Maxwell speed distribution in 3 dimension

$$f(V) = 4\pi\left(\frac{m}{2k\pi T}\right)^{3/2} V^2 e^{-\left(\frac{mV^2}{2kT}\right)}$$

$$\langle V^2 \rangle = \int_0^\infty V^2 f(V)\,dV$$

$$= \frac{3kT}{m}$$

$$V_{rms} = \sqrt{\langle V^2 \rangle} = \sqrt{\frac{3kT}{m}}$$

(i) for $T = 77k$, $\quad V_{rms} = \sqrt{\frac{3\times 1.38\times 10^{-23}\times 77}{2\times 14\times 1.66\times 10^{-27}}} = 262\ m/s$

(i) for $T = 300k$, $\quad V_{rms} = \sqrt{\frac{3\times 1.38\times 10^{-23}\times 300}{2\times 14\times 1.66\times 10^{-27}}} = 517\ m/s$

Problem 2:

n_0	n_1	n_2	n_3	n_4	n_5	n_6	
6						1	$W_1 = 7$
5	1				1		$W_2 = 42$
5		1		1			$W_3 = 42$
5			2				$W_4 = 21$
4	2			1			$W_5 = 105$
4	1	1	1				$W_6 = 210$
4		3					$W_7 = 35$
3	3		1				$W_8 = 140$
3	2	2					$W_9 = 210$
2	4	1					$W_{10} = 105$
1	6						$W_{11} = 7$

$$Z = \sum_i N \cdot W_i = 7 \times 924 = 6468$$

$$P(E_0) = \frac{\sum_i n_0 W_i}{Z} = \frac{1}{2}$$

$$P(E_1) = \frac{\sum_i n_1 W_i}{Z} = \frac{3}{11}$$

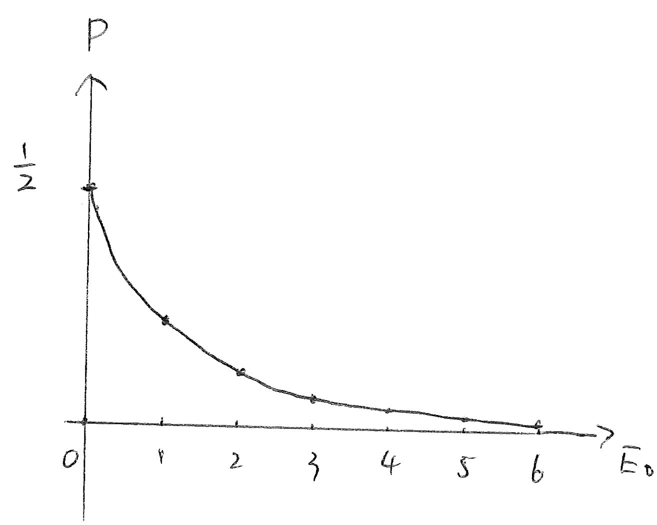
$$P(E_2) = \frac{3}{22}$$

$$P(E_3) = \frac{2}{33}$$

$$P(E_4) = \frac{1}{44}$$

$$P(E_5) = \frac{1}{154}$$

$$P(E_6) = \frac{1}{924}$$



(Note: This satisfies Boltzmann distribution)

Problem 3.

Maxwell speed distribution

$$f(v) = 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-\left(\frac{mv^2}{2kT}\right)}$$

$$\langle E_k \rangle = \langle \tfrac{1}{2} m v^2 \rangle = \tfrac{1}{2} m \langle v^2 \rangle$$

$$\langle v^2 \rangle = \int_0^\infty v^2 f(v) dv$$

$$= \frac{3kT}{m}$$

So $$\langle E_k \rangle = \frac{1}{2} m \cdot \frac{3kT}{m} = \frac{3}{2} kT$$

In this estimation, $\langle E_k \rangle \to 0$ when $T \to 0$, and thus $v^2 \to 0$

However, actual result is not like this. This is because eletrons are Fermions, which do not obey Boltzmann statistics, but obey Fermi statistics. All eletrons can not stay at ground state together, as a result, $\langle E_k \rangle \neq 0$ even when $T = 0$.