HW 6 Solution

1.5. (a).

$$\sigma T^4 = \frac{\Delta E}{At}$$

$$\Delta E = (5.67\times10^{-8}\frac{W}{m^2k^4})(5700k)^4\cdot\left(4\pi\left(\frac{1.4\times10^9}{2}m\right)^2\right)(1s)$$

$$= 3.69\times10^{26} J$$

$$\Delta E = \Delta m c^2$$

$$\Rightarrow \quad \Delta m = 4.09\times10^9 kg$$

(b).

$$\Delta E_{1year} = \Delta E_{1s}\times3600\times24\times365$$

$$\Delta m_{1year} = \Delta m_{1s}\times3600\times24\times365$$

$$= 1.29\times10^{17} kg$$

$$\frac{\Delta M_{1year}}{M_{sun}} = \frac{1.29\times10^{17}kg}{2\times10^{30}} = 6.45\times10^{-14}$$

1.8

$$\lambda_{max} T = 2.898\times10^{-3} m\cdot k$$

Human body temperature $\sim 310k$

$$\Rightarrow \quad \lambda_{max} = 9.35\times10^{-6} m$$

1.17.

$$R_T = \int_0^\infty \frac{2\pi h}{c^2}\frac{\nu^3 d\nu}{e^{h\nu/kT}-1}$$

set $\quad q = \frac{h\nu}{kT} \quad dq = \frac{h}{kT}d\nu$

$$R_T = \frac{2\pi h}{c^2}\left(\frac{kT}{h}\right)^4 \int_0^\infty \frac{q^3\,dq}{e^q - 1}$$

$$= \frac{2\pi h}{c^2}\left(\frac{kT}{h}\right)^4 \frac{\pi^4}{15}$$

$$= \frac{2\pi^5 k^4}{15c^2h^3} T^4$$

$$= \sigma T^4$$

4.15 In Bohr's model

$$L = n\hbar = n\frac{h}{2\pi}$$

Also,

$$L = mrv$$

$$E = -\frac{1}{2}mv^2 = -K \quad (K \text{ is kinetic energy})$$

$$\Rightarrow \quad v = \frac{2|E|r}{L}$$

$$T = \frac{2\pi r}{v}$$

$$\Rightarrow \text{frequency } \nu = \frac{1}{T} = \frac{v}{2\pi r} = \frac{2|E|}{2\pi L}$$

$$= \frac{2|E|}{nh}$$

4.17. (a) In Hydrogen atom $(Z=1)$ and ground state $n=1$.

we have:

$$\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} = m\frac{v^2}{r}$$

$$mrv = \hbar \quad (n=1)$$

$$\Rightarrow \quad v = \frac{e^2}{4\pi\varepsilon_0 mrv}$$

$$= \frac{e^2}{4\pi\varepsilon_0\hbar}$$

$$= \frac{e^2}{4\pi\varepsilon_0\hbar c}\cdot c$$

$$= \alpha c$$

(b). $\alpha = 7.30 \times 10^{-3}$

This means $\frac{v}{c} = \alpha = 7.30 \times 10^{-3}$

This velocity is relatively small comparing to speed of light.

So we can get reasonable results in Bohr's model even though we neglect relativistic effects.