HW5 Solution

13.1 (a).

$$E_p = \gamma m_p c^2$$

$$\Rightarrow \quad \gamma = \frac{10^{20}}{1.67\times10^{-27}\times(3\times10^8)^2/1.6\times10^{-19}} = 1.06\times10^{11}$$

$$\Rightarrow \quad V \approx 3\times10^8 \, m/s$$

Diameter of galaxy in proton's frame

$$D = D_0/\gamma$$

Propertime: $t = \frac{D}{V} = \frac{10^5}{1.06\times10^{11}}$ years ≈ 29.8 s

(b).

$$E_K = \frac{1}{2}mV^2 = \frac{1}{2}\times 0.145\times\left(\frac{1.609\times10^5}{3600}\right)^2 J$$

$$\approx 145 J$$

$$E_p = 10^{20}\times1.6\times10^{-19} J = 16 J$$

$E_K > E_p \qquad E_K/E_p \approx 9.06$

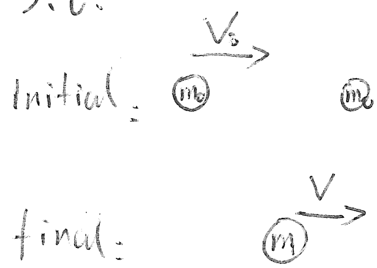
13.4 In one particle's rest frame, another particle has velocity.

$$V' = \frac{V+V}{1+\frac{V^2}{C^2}} = \frac{2V}{1+\frac{V^2}{C^2}}$$

The Energy is

$$E' = \frac{m_0 c^2}{\sqrt{1-\frac{V'^2}{C^2}}} = m_0 c^2\left(\frac{C^2+V^2}{C^2-V^2}\right)$$

13.6.



In this process, Momentum and energy are separately conserved:

From momentum conservation ($P_i = P_f$)

$$P_i = \gamma_0 m_0 V_0$$

$$P_f = \gamma m V$$

Because $P_i = P_f$

$$\Rightarrow \frac{m_0 V_0}{\sqrt{1-\frac{V_0^2}{c^2}}} = \frac{mV}{\sqrt{1-\frac{V^2}{c^2}}}$$

From energy conservation: ($E_i = E_f$)

$$E_i = m_0 c^2 + (m_0 c^2 + X m_0 c^2)$$

(total energy of one particle is sum of rest energy and kinetic energy $E = E_{kin} + E_0$)

$$E_f = \frac{mc^2}{\sqrt{1-\frac{V^2}{c^2}}}$$

Because $E_i = E_f$

$$\Rightarrow 2m_0 c^2 + X m_0 c^2 = \frac{mc^2}{\sqrt{1-\frac{V^2}{c^2}}}$$

Since we know.

$$E_f^2 - P_f^2 c^2 = m^2 c^4$$

$$E_1^2 - P_1^2 c^2 = m_0^2 c^4$$

where E_1 is the total energy of the moving intial particle, P_1 is the momentum of that particle, so

$$((1+X) m_0 c^2)^2 - P_1^2 c^2 = m_0^2 c^4$$

$$\Rightarrow \quad P_1 = \sqrt{(1+X)^2 - 1}\, m_0 c$$

Because $P_1 = P_i = P_f$ $\quad E_f = 2m_0 c^2 + X m_0 c^2$

$$\Rightarrow \quad (X+2)^2 m_0^2 c^4 - [(1+X)^2 - 1] m_0^2 c^4 = m^2 c^4$$

$$\Rightarrow \quad m = \sqrt{2X+4}\, m_0$$