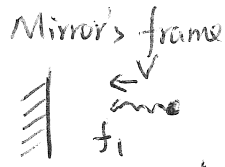
HW4 Solution

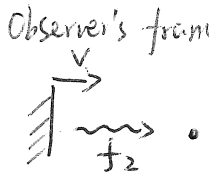
12.13.

In Mirror's rest frame, Incoming light has Doppler shift.



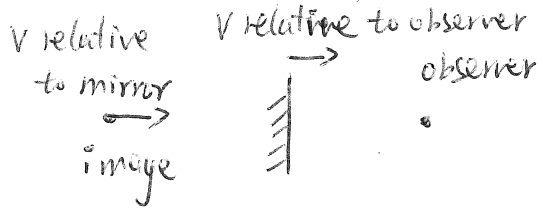
$$f_1 = f_0\sqrt{\frac{c+V}{c-V}}$$

After reflection, light has f_1 in mirror's frame. Since mirror is approaching the observer with V, so there is a second Doppler shift.



$$f_2 = f_1\sqrt{\frac{c+V}{c-V}} = f_0\sqrt{\left(\frac{c+V}{c-V}\right)^2} = f_0\left(\frac{c+V}{c-V}\right)$$

We can think of the total effect coming from the image source.



From Lorentz transformation, we can find the velocity of the image in observer's frame:

$$V' = \frac{V+V}{1+\frac{V^2}{c^2}} = \frac{2V}{1+\frac{V^2}{c^2}}$$

We can check the Doppler shift from the image is

$$f' = f_0\sqrt{\frac{c+V'}{c-V'}} = f_0\sqrt{\frac{(c+V)^2}{(c-V)^2}} = f_2$$

This matches previous result.

12.14. In A. B's rest frame, we can find:

Speed of light in glass $V_{in} = \dfrac{\frac{c}{n}+V}{1+\frac{c}{n}\frac{V}{c^2}}$

Length of glass slab : $D' = D\sqrt{1-\frac{V^2}{c^2}}$

Because glass slab is moving in the same direction of light, So the actual time that light is in glass (t_{in}) and the total distance that light travels in glass (L_{in}) are:

$$t_{in} = \frac{D'}{V_{in}-V} = D\,\frac{(1+\frac{V}{nc})}{\frac{c}{n}}\,\frac{1}{\sqrt{1-\frac{V^2}{c^2}}}$$

$$L_{in} = t_{in}V_{in} = D\,\frac{(\frac{c}{n}+V)}{\frac{c}{n}}\,\frac{1}{\sqrt{1-\frac{V^2}{c^2}}}$$

Since $L = L_{in} + L_{out}$, we can find

$$L_{out} = L - D\,\frac{(\frac{c}{n}+V)}{\frac{c}{n}}\,\frac{1}{\sqrt{1-\frac{V^2}{c^2}}}$$

Total time from A to B is

$$t = t_{in} + \frac{L_{out}}{c}$$

$$= \frac{L}{c} + \frac{D}{c}\frac{1}{\sqrt{1-\frac{V^2}{c^2}}}\left(n+\frac{V}{c}-1-\frac{V}{c}n\right)$$

$$= \frac{L}{c} + \frac{D}{c}(n-1)\sqrt{\frac{c-V}{c+V}}$$

12.15

(a). $c = \lambda \cdot f$

Doppler effect for wavelength:

$$\lambda = \lambda_0 \sqrt{\frac{c+V}{c-V}}$$

$$\lambda_0 = 6.561\times10^{-7}\,m \quad V = 3\times10^{6}\,m/s$$

So. $\lambda = 6.627\times10^{-7}\,m$

(b) λ_1 and λ_2 are wavelength from two ends of the sun.

$$\lambda_1 = \lambda_0 \sqrt{\frac{c+V'}{c-V'}}$$

$$\lambda_2 = \lambda_0 \sqrt{\frac{c-V'}{c+V'}}$$

$$\Delta\lambda = \lambda_0\left(\sqrt{\frac{c+V'}{c-V'}} - \sqrt{\frac{c-V'}{c+V'}}\right)$$

$$\approx \lambda_0\left(1+\frac{V'}{c} - \left(1-\frac{V'}{c}\right)\right)$$

$$\Delta\lambda \approx \lambda_0 \frac{2V'}{c}$$

$$V' = \frac{\Delta\lambda}{2\lambda_0} c = \frac{9\times10^{-12}}{2\times6.561\times10^{-7}} 3\times10^{8}\,m/s \approx 2.06\times10^{3}\,m/s$$

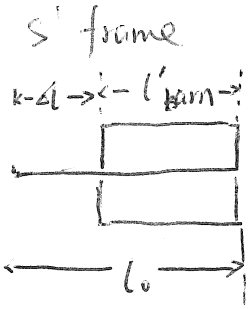
Period: $T = \frac{\pi D}{V'} = \frac{\pi\times1.4\times10^{9}\,m}{2.06\times10^{3}\,m/s} = 2.13\times10^{6}\,s$

≈ 24.7 days.

12.16 (a). From length contraction:

$$l = l_0\sqrt{1-\frac{v^2}{c^2}} = \frac{1}{2}l_0$$

(b) Length of the barn in S' frame is.



$$l'_{barn} = l_{barn}\sqrt{1-\frac{v^2}{c^2}} = \frac{3}{4}l_0 \times \frac{1}{2} = \frac{3}{8}l_0$$

$$\Delta l = l_0 - l'_{barn} = \frac{5}{8}l_0$$

So in S' frame, B is $\frac{5}{8}l_0$ away from the front door.

(c). Because $l_0 > l'_{barn}$ in S' frame, so A. B do not lie inside the barn at the same instant.