

HW10 Solution

Problem 1:

(a).

$$\Delta Q = C \Delta T$$

$$\Delta Q = 300 \times (400 - 350) = 15000 \text{ J}$$

(b)

$$\Delta S_1 = C \ln\left(\frac{350}{300}\right) = 46.25 \text{ J/K}$$

$$\Delta S_2 = C \ln\left(\frac{350}{400}\right) = -40.06 \text{ J/K}$$

(c)

$$\Delta S = \Delta S_1 + \Delta S_2 = 6.19 \text{ J/K} > 0$$

Problem 2:

$$H = -\sum_i P_i \log_2 P_i$$

Summing over all letters:

$$H_E = 4.162$$

$$H_G = 4.096$$

English has

$H_E > H_G \Rightarrow \wedge$ More information per letter than German.

\Rightarrow German words should contain more letters to have the same information as English words

Problem 3:

(a).

$$P_i = 1 \Rightarrow H = -\sum_i P_i \log_2 P_i = -1 \times \log_2 1 = 0$$

(b).

$$P_i = \frac{1}{52!} \Rightarrow H = -\sum_{i=1}^{52!} P_i \log_2 P_i = \log_2 52! = 225.6$$

(C) Assuming each shuffle has the same contribution to entropy.

$$\Delta H = \frac{225.6}{9} = 25$$

Problem 4:

(a).

in put		out put	
A	B	A'	B'
1	1	0	0
1	0	0	1
0	1	1	0
0	0	1	1

$$H_{in} = -\sum_i P_i \log_2 P_i$$

$$P_i = \frac{1}{4}$$

$$\Rightarrow H_{in} = 2$$

$$\Rightarrow \Delta H = H_{out} - H_{in} = 0$$

$$H_{out} = -\sum_i P_i \log_2 P_i$$

$$P_i = \frac{1}{4}$$

$$\Rightarrow H_{out} = 2$$

(b).

in put		out put	
A	B	A'	B'
1	1	1	1
1	0	1	0
0	1	1	0
0	0	0	0

$$H_{in} = -\sum_i P_i \log_2 P_i$$

$$P_i = \frac{1}{4}$$

$$\Rightarrow H_{in} = 2$$

$$H_{out} = -\sum_i P_i \log_2 P_i$$

$$P_1 = \frac{1}{4} \quad P_2 = \frac{1}{2} \quad P_3 = \frac{1}{4}$$

$$\Rightarrow H_{out} = 1.5$$

$$\Rightarrow \Delta H = H_{out} - H_{in} = -0.5$$