

What happens when two qubits prepared in an entangled “singlet” quantum state

$|\Psi\rangle = \frac{|0\rangle|1\rangle - |1\rangle|0\rangle}{\sqrt{2}}$ are both rotated with the same angle θ and phase ϕ :

$$R(\theta, \phi) |0\rangle = \cos\left(\frac{\theta}{2}\right) |0\rangle + e^{i\phi} \sin\left(\frac{\theta}{2}\right) |1\rangle$$

$$R(\theta, \phi) |1\rangle = \cos\left(\frac{\theta}{2}\right) |1\rangle - e^{-i\phi} \sin\left(\frac{\theta}{2}\right) |0\rangle$$

$R_1(\theta, \phi) R_2(\theta, \phi) |\Psi\rangle$

$$= \frac{1}{\sqrt{2}} \left(\cos\left(\frac{\theta}{2}\right) |0\rangle + e^{i\phi} \sin\left(\frac{\theta}{2}\right) |1\rangle \right) \left(\cos\left(\frac{\theta}{2}\right) |1\rangle - e^{-i\phi} \sin\left(\frac{\theta}{2}\right) |0\rangle \right)$$

$$- \frac{1}{\sqrt{2}} \left(\cos\left(\frac{\theta}{2}\right) |1\rangle - e^{-i\phi} \sin\left(\frac{\theta}{2}\right) |0\rangle \right) \left(\cos\left(\frac{\theta}{2}\right) |0\rangle + e^{i\phi} \sin\left(\frac{\theta}{2}\right) |1\rangle \right)$$

$$= \frac{|0\rangle|1\rangle - |1\rangle|0\rangle}{\sqrt{2}}$$

doesn't change!!!