Phys 371 Quantum Lecture 6 3/25/2016

Agenda

1. Quantum Probability + Measurement
2. Wavefunction - Properties
   a. Normalization
   b. observable operators

The wavefunction:

The quantum state $\Psi(x,t)$ evolves acording to the schrödinger equation (SE)

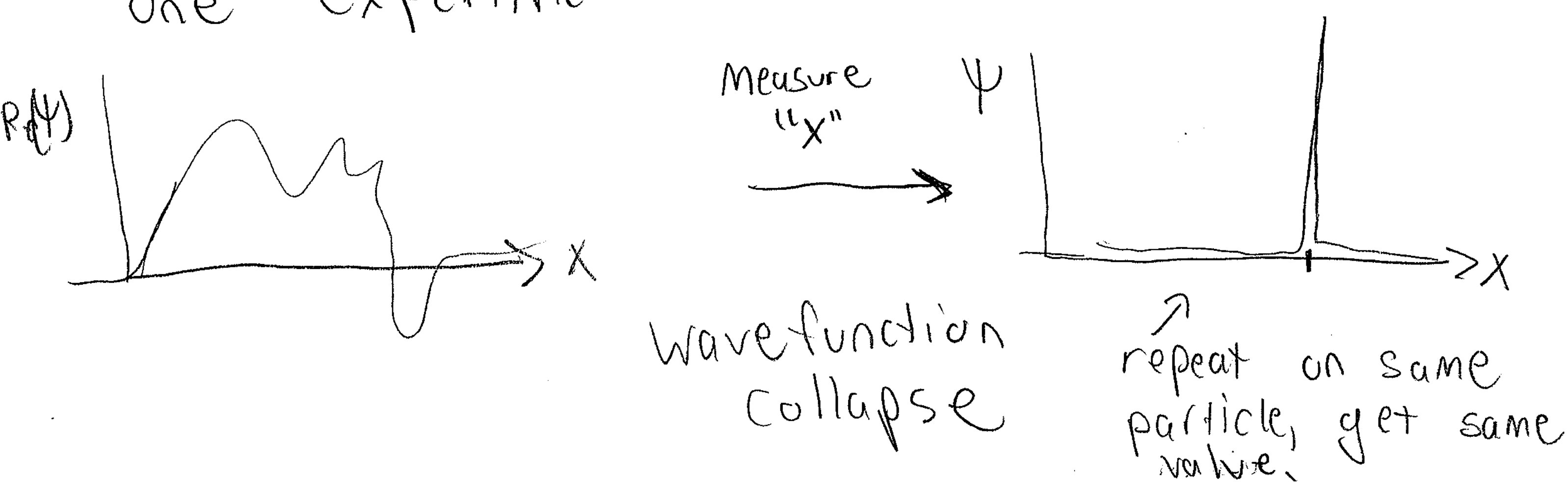
$$\frac{\partial \Psi(x,t)}{\partial t} = \frac{i\hbar}{2m}\frac{\partial^2 \Psi(x,t)}{\partial x^2} - \frac{i}{\hbar} V(x)\Psi(x,t)$$

The wavefunction is a tool for calculation!

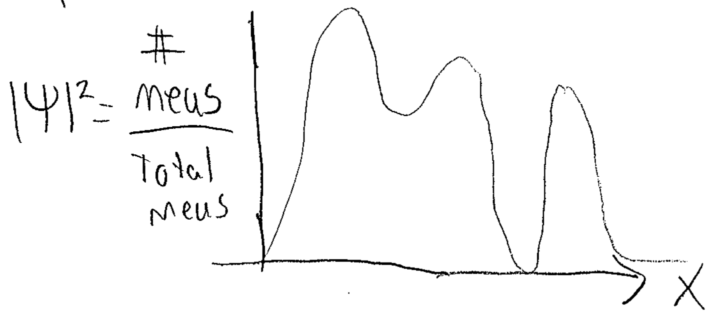
- Physically Relevant Quantities are related to

  $|\Psi^* \Psi| \rightarrow$ Probability Density

- How do we measure a quantum variable? one experiment at a time!



Only from repeated measurements can I construct a probability distribution for where to find the particle.



Return to Double Slit experiment using phet simulation. See $|\psi|^2$ built up from repeated measurements.

Normalization:

A probability distribution must be normalized

$$\int_{-\infty}^{\infty} \psi^* \psi \, dx = 1$$

$|\psi(x,t)|^2 \Rightarrow$ units $= 1/L$

Why?: $\int_a^b \psi^* \psi \, dx =$ Prob to find particle between a and b

Probability to find it anywhere in space must be 1.

The Schrodinger Equation Preserves Normalization

→ In other words, probability does not leak away as a function of time.

Proof: Griffiths 1.4 (If time...)

$$\frac{d}{dt} \int_{-\infty}^{\infty} |\psi(x,t)|^2 dx = \int_{-\infty}^{\infty} \frac{\partial}{\partial t} (\psi^* \psi) \, dx$$

$$= \int_{-\infty}^{\infty} \left(\psi^* \frac{\partial \psi}{\partial t} + \frac{\partial \psi^*}{\partial t} \psi \right) dx$$

Product Rule

→ Put in $\frac{\partial \psi}{\partial t}$, $\frac{\partial \psi^*}{\partial t}$ from SE

$$= \int_{-\infty}^{\infty} \frac{i\hbar}{2m}\frac{\partial^2\psi}{\partial x^2}\psi^* - \frac{i}{\cancel{\hbar}}\cancel{V}\psi\psi^* + \left(-\frac{i\hbar}{2m}\frac{\partial^2\psi^*}{\partial x^2}\psi + \frac{i}{\cancel{\hbar}}\cancel{V}\psi^*\psi\right)$$

$$= \frac{\partial}{\partial x}\int_{-\infty}^{\infty} \frac{i\hbar}{2m}\left[\psi^*\frac{\partial\psi}{\partial x} - \frac{\partial\psi^*}{\partial x}\psi\right]dx$$

$$= \frac{i\hbar}{2m}\left(\psi^*\frac{\partial\psi}{\partial x} - \frac{\partial\psi^*}{\partial x}\psi\right)\Big|_{-\infty}^{\infty}$$

We require $\psi(x=\pm\infty) = 0$ therefore

required for normalization.

* Pathological counterexamples need not apply...

$$\boxed{\frac{d}{dt}\int_{-\infty}^{\infty}\psi^*(x,t)\,\psi(x,t)\,dx = 0}$$

Measuring Averages (Expectation Values)

Position: $\langle x\rangle = \int_{-\infty}^{\infty} x|\psi(x,t)|^2 dx$

Equivalent to a weighted average since $|\psi(x,t)|^2$ is a probability density

True for any $f(x)$

$$\langle f(x)\rangle = \int_{-\infty}^{\infty} f(x)|\psi(x,t)|^2 dx$$

Momentum: Expectation value of p should follow classical dynamics $\langle p\rangle = m\frac{d\langle x\rangle}{dt}$

$$\langle p\rangle = \frac{i\hbar}{2}\int_{-\infty}^{\infty} x\frac{\partial}{\partial x}\left(\psi^*\frac{\partial\psi}{\partial x} - \frac{\partial\psi^*}{\partial x}\psi\right)dx$$

Integrate by Parts twice

$$\langle p \rangle = -i\hbar \int_{-\infty}^{\infty} \psi^* \frac{\partial \psi}{\partial x}\, dx$$

See Griffiths 1.5

Expectation values of momentum come from <u>spatial derivitives</u> of $\psi(x,t)$

<u>Definition</u>: $\underset{\text{position}}{x}$, $\underset{\text{momentum}}{-i\hbar \frac{\partial}{\partial x}}$, or $f(x, -i\hbar \frac{\partial}{\partial x})$ are <u>operators</u>

correspond to measurements of observables

<u>Operator</u>: $O(x,p)$

<u>Average Value</u>: $\langle O(x,p) \rangle = \int_{-\infty}^{\infty} \psi^*(x,t)\, O(x,p)\, \psi(x,t)\, dx$

A thing we can measure in an experiment.

<u>(Alternate) Momentum</u>: We know $\psi = Ae^{i(kx-\omega t)}$ solves ~~SE~~ E.

Note that $-i\hbar \frac{\partial \psi}{\partial x} = -i^2 \hbar k A e^{i(kx-\omega t)}$

$$= \hbar k \psi$$

$$= p\psi$$

Operator $-i\hbar \frac{\partial \psi}{\partial x} \Rightarrow$ momentum p of state ψ

I assert that

$$\langle p \rangle = \int_{-\infty}^{\infty} \psi^*(x,t)\, p\, \psi(x,t)\, dx = -i\hbar \int_{-\infty}^{\infty} \psi^*(x,t) \frac{\partial \psi}{\partial x}(x,t)\, dx$$