Time - Energy Uncertainty

$$\Delta t \cdot \Delta E \geq \hbar/2$$

Time for the system to evolve. If it never evolves → constant E.

Time Dependent Schrödinger Eq.

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} + V\Psi$$

Separation of Variables (See Griffiths 2.1)

$$\Psi(x,t) = \psi(x)\,\varphi(t)$$

$$\frac{\partial \Psi}{\partial t} = \psi \frac{\partial \varphi}{\partial t}, \qquad \frac{\partial^2 \Psi}{\partial x^2} = \frac{\partial^2 \psi}{\partial x^2}\varphi$$

$$i\hbar\,\psi \frac{\partial \varphi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2}\varphi + V\psi\varphi$$

$$i\hbar \frac{1}{\varphi}\frac{\partial \varphi}{\partial t} = -\frac{\hbar^2}{2m}\frac{1}{\psi}\frac{\partial^2 \psi}{\partial x^2} + V$$

constant = constant (call it E)

t dependent

$$i\hbar \frac{1}{\varphi}\frac{d\varphi}{dt} = E$$

$$\frac{d\varphi}{dt} = -\frac{iE}{\hbar}\varphi$$

$$\boxed{\varphi(t) = e^{-iEt/\hbar}}$$

time dependent phase

x dependent

$$\boxed{-\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + V\psi = E\psi}$$

$$\left(\frac{p^2}{2m} + V\right)\psi = E\psi$$

$$\boxed{\hat{H}\psi = E\psi}$$

Hamiltonian operator (total energy)

Time Independent Schrödinger Eq.

Stationary States

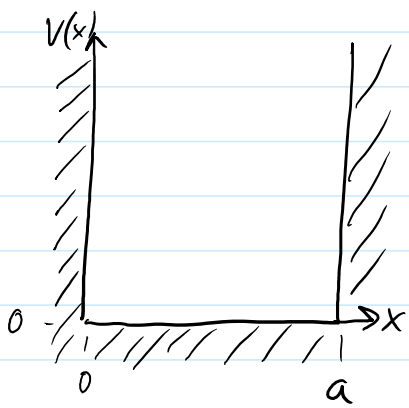
$$|\Psi(x,t)|^2 = \int \psi^* e^{+iEt/\hbar} \psi e^{-iEt/\hbar} dx = |\psi(x)|^2$$

$$\sigma_H^2 = \langle H^2 \rangle - \langle H \rangle^2 = E^2 - E^2 = 0$$

General Solution

$$\Psi(x,t) = \sum_{n=1}^{\infty} c_n \psi_n(x) e^{-iE_n t/\hbar}$$

Example potential: Infinite Square Well (See Griffiths 2.2)



$$V(x) = \begin{cases} 0 & \text{if } 0 \le x \le a \\ \infty & \text{otherwise} \end{cases}$$

Outside well: $\psi(x) = 0$ to cancel $V = \infty$ and allow normalization

Inside well: $V = 0 \rightarrow \frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} = E\psi$

let $k \equiv \frac{\sqrt{2mE}}{\hbar}$ $\qquad \frac{d^2\psi}{dx^2} = -k^2 \psi$

classical harmonic oscillator! Hooke's Law!

solutions: $\psi(x) = A \sin kx + B \cos kx$

Boundary conditions (and continuity)

at $x=0$: $\psi(0)=0 = A\sin 0 + B\cos 0 \rightarrow B=0$

$\therefore \psi(x) = A\sin kx$

at $x=a$: $\psi(a)=0 = A\sin ka \rightarrow ka = 0, \pm\pi, \pm 2\pi$,

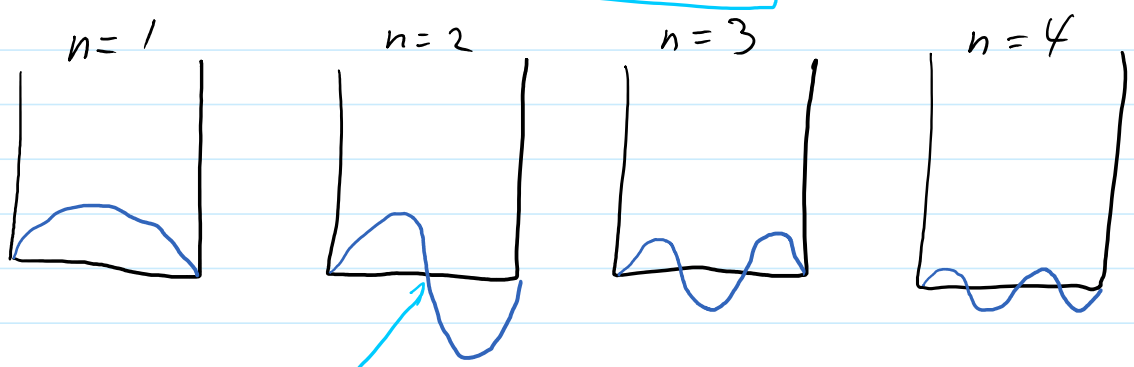
negative ka are repeat solutions, and $ka=0$ is not normalizable.

$$K_n = \frac{n\pi}{a} \quad : n = 1, 2, 3, \ldots \qquad E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}$$

quadratic

Normalize to get $A = \sqrt{\frac{2}{a}}$

$$\boxed{\psi_n(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi}{a}x\right)}$$



This is the expected position, but in $n \in$ even modes there is zero probability of finding the particle there!

The discrete states are a result of the particle being bound in a potential. At last we have seen something quantized about quantum mechanics.