Physics 371 Exam #2 April 11, 2016

Planck's constant $h = 6.626 \times 10^{-34}$ kg m^2/s, or $\hbar = h/2\pi = 1.055 \times 10^{-34}$ kg m^2/s

Time dependent Schrödinger equation: $i\hbar \frac{\partial \Psi(\mathrm{x,t})}{\partial t} = -\frac{\hbar}{2m}\frac{\partial^2 \Psi(\mathrm{x,t})}{\partial x} + V(x)\Psi(\mathrm{x,t})$

General solution $\Psi(\mathrm{x,t}) = \sum_n c_n \,\psi_n(x)\, e^{-iE_n t/\hbar}$

Time independent Schrödinger equation for states with energy E: $-\frac{\hbar}{2m}\frac{\partial^2\psi}{\partial x} + V(x)\psi = E\psi$

Infinite square well (width a) energies $E_n = \left(\frac{\pi^2\hbar^2}{2ma^2}\right) n^2$, where $n = 1, 2, 3, \ldots$

Lowest (ground) state wavefunctions $\psi_{n=0}(x)$ of the harmonic oscillator, with $x_0 = \sqrt{\frac{\hbar}{2m\omega}}$

$$\psi_0(x) = \left(\frac{1}{\sqrt{2\pi}x_0}\right)^{\frac{1}{2}} e^{-x^2/(4x_0^2)}$$

1. Fun Quick Problems

(a) [5] We derived the condition for the circular Bohr orbits of an electron around its nucleus that the circumference must be an integral multiple n of the deBroglie wavelength $\lambda = h/p$. What is the equivalent condition on the values of angular momentum $\boldsymbol{L} = \boldsymbol{r} \times \boldsymbol{p}$ of the electron?

For circular motion, **r** is perpendicular to **p**, so $\mathrm{L = rp = \frac{n}{2\pi}\left(\frac{h}{p}\right) p = n\left(\frac{h}{2\pi}\right)}$

(b) [5] What are the units of the wavefunction of a particle in 1 spatial dimension? Justify your answer.

$$\int_{-\infty}^{+\infty} |\varphi(x)|^2 dx = 1, \text{ so } \varphi(x) \propto \frac{1}{\sqrt{length}}$$

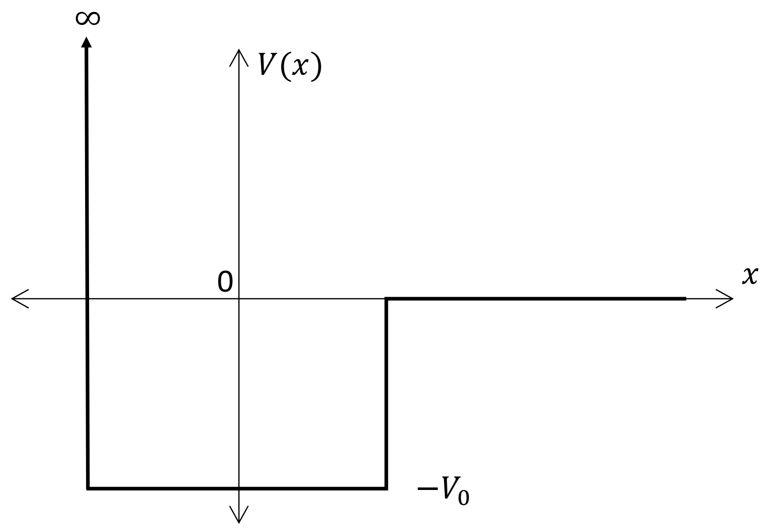
(c) [5] A particle has a pretty well-defined momentum, with uncertainty $\Delta p = \sqrt{\langle p^2\rangle - \langle p\rangle^2} = 5.275 \times 10^{-29}$ kg·m/s. What is the minimum uncertainty of the particle's position $\Delta x = \sqrt{\langle x^2\rangle - \langle x\rangle^2}$?

$$\Delta p \Delta x \leq \frac{\hbar}{2}, \text{ so } \Delta x \leq \frac{\hbar}{2\Delta p} = \frac{1.055\times10^{-34}}{2\times5.275\times10^{-29}} = 10^{-6} m$$

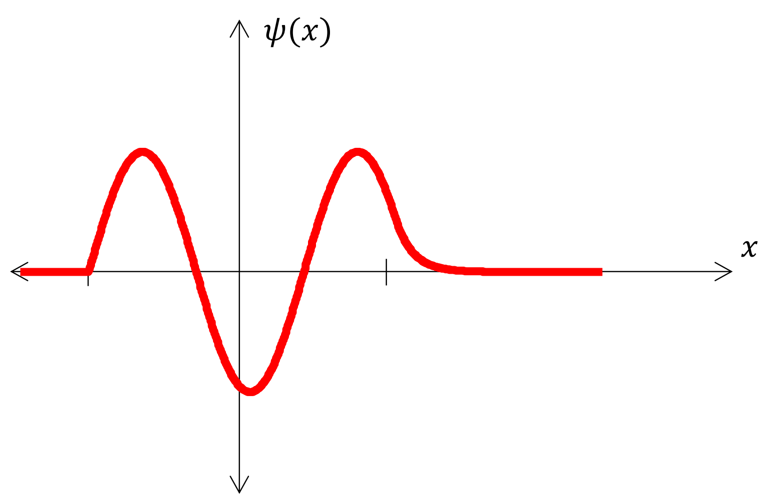
2. Quantum Mechanics in 1D potentials

(a) [5] A massive particle moves in 1D and is subjected to the potential energy function $V(x)$ plotted below. Mark the table with an "X" concerning states with energy E indicated.

Energy range	Any value OK	Quantized values only	No values allowed
$E < -V_0$			X
$-V_0 < E < 0$		X	
$E > 0$	X		



(b) [10] For the same 1D potential energy function in (a), a particle has energy E in the range $-V_0 < E < 0$. Sketch a possible wavefunction $\psi(x)$ for this stationary state in the graph provided. (Make sure to line up the x-axis with the potential function above!)



3. Measuring bound particles

(a) [10] A particle of mass m is in the ground state of a 1D harmonic oscillator with energy $\hbar\omega/2$. Find the probability that an observation of the position x of this particle will detect the particle in the classically forbidden region where $\frac{1}{2}m\omega^2x^2 > E$.

HINT: $\int_0^1 e^{-z^2}dz \approx 0.75$

$$P_{\text{forbidden}} = 1 - \int_{-x_c}^{+x_c} |\psi_0(x)|^2 dx \quad \text{where} \quad x_c = \sqrt{\frac{2E}{m\omega^2}} = \sqrt{\frac{\hbar}{m\omega}} = x_0\sqrt{2}$$

$$= 1 - \frac{1}{x_0\sqrt{2\pi}} \int_{-x_0\sqrt{2}}^{x_0\sqrt{2}} e^{-x^2/2x_0^2} dx = 1 - \frac{1}{\sqrt{\pi}} \int_{-1}^{1} e^{-z^2} dz \qquad z = \frac{x}{x_0\sqrt{2}}$$

$$= 1 - \frac{2}{\sqrt{\pi}} \int_0^1 e^{-z^2} dz = 1 - \frac{2}{\sqrt{\pi}}(0.75) = \underline{0.154}$$

(b) [5] A particle of mass m is confined in an infinite square well of width a and prepared in a state of definite energy $E = \left(\frac{\pi^2\hbar^2}{2ma^2}\right)n^2$ where n is a nonzero integer. Its position is then measured and found to be precisely in the middle of the well, a distance $a/2$ from each wall. What can you say about the possible values of n ?

n must be odd (for n even, $\varphi(middle) = 0$)