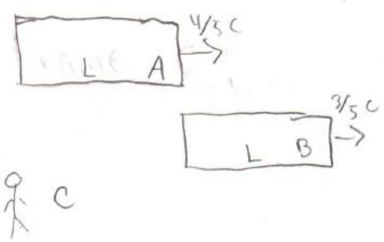
Physics 371 Exam #1 March 4, 2016

1. (30 points) Consider two trains traveling in the same direction on parallel tracks. Train A with speed $\left(\frac{4}{5}\right) c$ is about to overtake Train B with speed $\left(\frac{3}{5}\right) c$, as seen by an observer on the ground (Frame C). Both trains have proper (rest frame) length L. Consider the two events that describe the trains passing:

   - E_1 when the front of A passes the back of B
   - E_2 when the back of A passes the front of B.

   These events $(E_2 - E_1)$ have a separation in time Δt and separation in space Δx. Fill in the following table describing the train passing events in the three frames

	Frame A	Frame B	Frame C
Δt	*5L/c*	*5L/c*	*7L/c*
Δx	*–L*	*L*	*5L*



From C Frame:

$$\gamma_A = \frac{1}{\sqrt{1-16/25}} = \frac{5}{3}$$

$$\gamma_B = \frac{1}{\sqrt{1-9/25}} = \frac{5}{4}$$

Both trains are length contracted $L_A' = \frac{L}{\gamma_A} = \frac{3L}{5}$

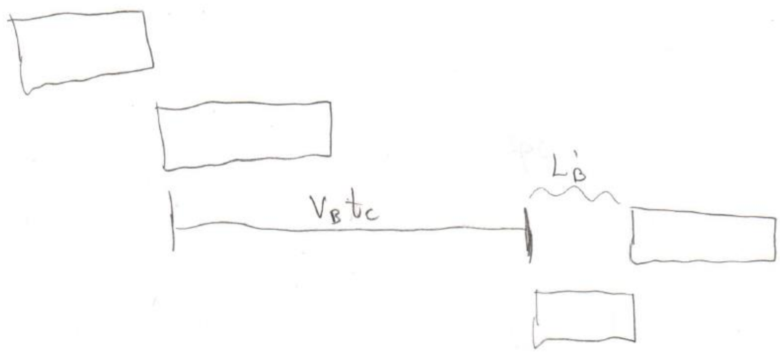
$L_B' = \frac{L}{\gamma_B} = \frac{4L}{5}$

The time interval is given by the length of time it takes A to travel the combined length of the two trains, plus the distance train B has traveled.

$$\Delta t_C = \frac{L_A'}{V_A} + \frac{L_B'}{V_A} + \frac{\Delta t_C V_B}{V_A} \Rightarrow \Delta t_C = \frac{L_A' + L_B'}{V_A - V_B}$$

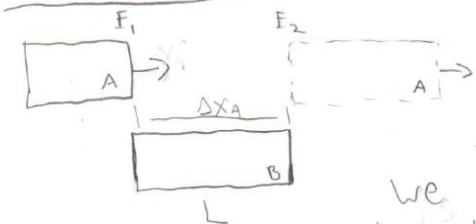
$$\Delta t_C = \frac{3L/5 + 4L/5}{4/5\,c - 3/5\,c} = \frac{7L/5}{1/5\,c} = 7\,L/c$$

The two events are spaced in frame C by



$$\Delta X_C = L_B' + V_B \Delta t_c = \frac{4L}{5} + \frac{3}{5}c\left(\frac{7L}{c}\right) = \frac{4L}{5} + \frac{21L}{5} = 5L$$

In Frame B:



The length must be the proper length of train B.

$$\Delta X_B = L$$

We can confirm this with a Lorentz transformation boosted into the frame of train B

$$\Delta X_B = \gamma_B(\Delta x_C - V_B \Delta t_c) = \frac{5}{4}\left(5L - \left(\tfrac{3}{5}c\right)\left(\tfrac{7L}{c}\right)\right) = \frac{5}{4}\left(\frac{25}{5}L - \frac{21}{5}L\right)$$

$$= \frac{5}{4}\left(\frac{4}{5}L\right) = L \quad \checkmark$$

The time interval is also given by a LT

$$\Delta t_B = \gamma_B(\Delta t_c - V_B \Delta x_c / c^2) = \frac{5}{4}\left(\frac{7L}{c} - \left(\frac{3}{5}c\right)(5L)/c^2\right)$$

$$= \frac{5}{4}(4L/c) = 5\,L/c$$

In Frame A: Again use Lorentz Transformations

$$\Delta t_A = \gamma_A(\Delta t_c - V_A \Delta x_c / c^2) = \frac{5}{3}\left(7L/c - \left(\tfrac{4}{5}c\right)5L/c^2\right) = \frac{5}{3}\left(\frac{3L}{c}\right)$$

$$= 5\,L/c$$

$$\Delta X_A = \gamma_A(\Delta t_c - V_A \Delta t_c) = \frac{5}{3}\left(5L - \tfrac{4}{5}c\left(\tfrac{7L}{c}\right)\right)$$

$$= \frac{5}{3}\left(\frac{25}{5}L - \frac{28}{5}L\right) = \frac{5}{3}\left(\frac{-3}{5}L\right)$$

$$\Delta X_A = -L$$

2. (15 points) A relativistic particle is measured in a certain inertial reference frame to have a total energy of 5 GeV (that's 5×10^9 eV) and a momentum of 3 GeV/c

a) What is the mass of the particle, in GeV/c^2?

$\vec{p}\cdot\vec{p} = m^2c^2 \Rightarrow E^2 - p^2c^2 = m^2c^4$

$m^2 = \frac{E^2}{c^4} - \frac{p^2}{c^2} = \frac{25\ \text{GeV}^2}{c^4} - \frac{9\ \text{GeV}^2}{c^4} = 16\ \frac{\text{GeV}^2}{c^4} \Rightarrow \boxed{m = 4\ \text{GeV}/c^2}$

b) What is the velocity of the particle, v/c?

$E = \gamma mc^2 \Rightarrow \gamma = \frac{5\ \text{GeV}}{4\ \text{GeV}} = 5/4 = \frac{1}{\sqrt{1-v^2/c^2}}$

$1 - \frac{v^2}{c^2} = \frac{16}{25} \Rightarrow \frac{v}{c} = \sqrt{1-\frac{16}{25}} = \sqrt{\frac{25-16}{25}} = \sqrt{\frac{9}{25}} = \frac{3}{5}.\ \boxed{\frac{v}{c} = 3/5}$

c) What is the energy of the particle in an IRF in which the particle's momentum is 4 GeV/c?

$E^2 - p^2c^2 = m^2c^4 \quad E = \sqrt{m^2c^4 + p^2c^2}$

$E = \sqrt{(4\ \text{GeV}/c^2)^2c^4 + (4\ \text{GeV}/c)^2c^2} = \sqrt{32}\ \text{GeV} = \boxed{4\sqrt{2}\ \text{GeV}}$

d) What is the kinetic energy of the particle in this new IRF?

$KE = mc^2(\gamma - 1) = E - mc^2 = 4\sqrt{2}\ \text{GeV} - 4\ \text{GeV}$

$= \boxed{4(\sqrt{2}-1)\ \text{GeV}}$

e) What is the maximum momentum this particle can have, according to the limits set by special relativity?

SR places no upper limit on momentum.

3. (5 points) A photon of energy E collides with a stationary particle of rest mass m_0 and is absorbed. Express the velocity of the resulting composite system in terms of m_0, E, & c.

Total energy is conserved, so $E + m_0c^2 = m_0\gamma c^2 = \frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}}$, where v is the final velocity.

Solving for v, we find $v = \sqrt{1 - \frac{1}{(1+\varepsilon)^2}}\,c = \left(\frac{\sqrt{\varepsilon(2+\varepsilon)}}{1+\varepsilon}\right)c$, where $\varepsilon = \frac{E}{m_0c^2}$