STATE DETECTION OF A TRAPPED ION QUBIT USING PHOTON ARRIVAL TIMES

by

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ABSTRACT

STATE DETECTION OF A TRAPPED ION QUBIT USING PHOTON ARRIVAL TIMES

by

Kenneth William Lee

Chair: Christopher Monroe

Quantum information processing is a field of science that results from using the physical reality that is described by quantum mechanics to perform computational tasks that were previously regarded as impossible or infeasible. Systems of trapped atomic ions have shown to be ideal candidates for the realization of a quantum computer because of their long trapping times, superior coherence properties and complete control of internal atomic states. A major tenet of universal quantum computation is the efficient readout of the state of a qubit. State detection in trapped atomic ions is most commonly accomplished through a state-dependent fluorescence method. In this work, I present a scheme that utilizes the arrival times of photons collected during the detection period and employs the principle of maximum likelihood to determine the state of the ion [1, 2]. The qubit is stored in the hyperfine levels of a single Ytterbium (Yb⁺) ion and the state of the ion is accurately measured with a fidelity of 98.9%. The method demonstrated here accounts for several sources of error that occur in previous state detection methods, and is a promising step toward the realization of fault-tolerant quantum computation.

CHAPTER I

Introduction

"Logic will get you from A to B. Imagination will take you everywhere."

-Albert Einstein

The field of quantum information science has significant implications in the fields of computer science and communication. Quantum information science uses the physics described by quantum mechanics in order to address problems in computation and communication that were previously thought to be impossible or infeasible. It has been shown that quantum computation has the ability to perform some tasks far more efficiently than any classical computation and that individual quantum systems may be used as quantum bits (qubits) for computation [3, 4]. The advent of powerful quantum algorithms, notably Shor's algorithm and Grover's Search algorithm, has catapulted the study of quantum information to the forefront of cryptography and secure communication [5, 6]. The exotic nature of quantum systems is what allows the quantum computer to be a powerful tool.

The basic requirements to achieve universal quantum computation have been outlined by David DiVincenzo, and among those requirements is efficient qubit readout. Due to the fragile nature of a quantum state, the readout of a qubit is not quite as cut and dry as the readout of a classical bit. Decoherence and decay, among other sources of noise, can perturb the quantum system and affect qubit readout, having significant consequences. Moreover, state detection of a qubit is important for what is called fault-tolerant computation, that is, computation that allows encoding and error recovery in such a way that the encoding and recovery schemes themselves are robust against errors. Therefore, efficient state detection is necessary in order to correctly implement quantum algorithms and error correction schemes [7].

Atomic ions have proven to be a promising candidate for the realization of quantum computation.

Their long trapping times, superior coherence properties as well as their control of internal states make ions very pure qubits [8]. Furthermore, trapped ions have been used to easily generate quantum entanglement with high fidelity, which is a critical requirement for many quantum computational protocols [9]. There are several proposed schemes for building a quantum computer using trapped ions. The first quantum computing scheme was proposed by Cirac and Zoller in 1995 [10] and uses a linear chain of ions in a single trap. Another scheme, proposed by Kielpinski, Monroe, and Wineland [11] utilizes an array of traps, in which these traps can be used as channels for shuttling ions throughout the array. Using a large array of traps allows certain regions of the array to be used for different purposes, such as memory and interaction zones.

In general, a qubit is stored in the hyperfine states of an ion or in an optical transition between some ground state and a metastable state. This thesis focuses on the hyperfine clock qubit in the ground state of Ytterbium. State detection of a hyperfine qubit is accomplished through a state dependent fluorescence method in which a cycling transition is resonantly driven with one of the qubit states.

In the following chapters of this thesis, I provide a procedure used to accomplish accurate qubit readout by using the arrival times of photons during the detection period and using the principle of maximum likelihood to infer the state of the qubit. In chapter 2, I present a brief overview of quantum computation, including a discussion of quantum algorithms and universal quantum computation. In the following chapter, I introduce the Ytterbium ion and discuss its atomic properties as well as all of the mechanisms used to trap and cool the ion. Next, we delve into two specific state detection methods: a discrimination method, which has been used previously in this experiment – and a method employing the maximum likelihood principle. In chapter 5, I present results from simulations of random photon generation along with theoretical errors associated with both the discriminator and maximum likelihood detection schemes. Chapter 6 offers experimental data and analysis using both of the methods described in chapter 5. We conclude with chapter 7 in which I provide an overview of the implications of the maximum likelihood method in quantum computation.

CHAPTER II

Quantum Computation

Speaker: "And the winner is ... Number 3, in a quantum finish."

"No fair! You changed the outcome by measuring it!"

-Professor Hubert J. Farnsworth (Futurama)

2.1 The Quantum Bit

A quantum bit (qubit) is the fundamental building block of a quantum computer. While the classical binary digit, or bit, can only assume a value of 0 or 1, a qubit can be in any arbitrary superposition of the states 0 and 1,

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \tag{2.1}$$

where α and β are two arbitrary complex numbers such that $|\alpha|^2 + |\beta|^2 = 1$. The fact that a quantum bit can be in a superposition of states allows for dense encoding, making quantum computation a powerful tool. If one were to expand the system to 2 classical bits, then there would be 4 possible states: 00, 01, 10, 11. Consequently, two qubits may simultaneously assume all 4 of these states, as so:

$$|\psi\rangle = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle \tag{2.2}$$

In general, for a system of N qubits, there are 2^N possible states that may be assumed simultaneously. Quantum computation takes advantage of this potentially dense encoding to perform deterministic information processing. An interesting phenomenon that arises out of this principle of superposition is entanglement. Entanglement represents a correlation between two quantum systems with ill-defined properties. Mathematically, it occurs when you cannot write the total quantum state of a system in terms of a product of the states of each individual qubit. To see this more clearly, consider a 2 qubit system in the following state:

$$|\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle|0\rangle + \frac{1}{\sqrt{2}}|0\rangle|1\rangle$$
(2.3)

Notice that this can simply be written as a product of the first qubit state and the second qubit state:

$$|\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle(|0\rangle + |1\rangle) \tag{2.4}$$

Now, consider the following wave function:

$$|\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle|0\rangle + \frac{1}{\sqrt{2}}|1\rangle|1\rangle \tag{2.5}$$

There is simply no way to write this as a product of the individual qubit states, so the two qubits are entangled. What makes entanglement such an interesting phenomenon is that if one were to make a measurement of one of the qubits, then something is automatically known about the other qubit. For instance, in the case described by 2.5, if a measurement were to be made of the first qubit, there is a 50 percent chance that the qubit reads out 0 or 1. But notice that if the first qubit is measured to be 1, then we know without even measuring the second qubit that the second qubit will also be in state 1! What makes this truly a unique, quantum phenomenon is that before a measurement is made, the system has a well defined state given by 2.5; however, each individual qubit does not have a well defined state! Entanglement has proven to be an extremely important resource in quantum information theory. Moreover, entanglement has been easily generated in trapped atomic ions, which are ideal for quantum hardware since they are natural carriers of quantum information.

2.2 Quantum Algorithms

Quantum computers have the potential to do things that are otherwise impossible or impractical on a classical computer. In fact, it has been shown by David Deutsch that a quantum computer can efficiently simulate any realistic model of computation. Richard Feynman even theorized that the quantum computer can even simulate quantum physics, such as Heisenberg-like spin Hamiltonians. Some of the power of a quantum computer can be seen in this simple example: if a quantum computer were to have 300 qubits, then there would be a possible 2^{300} states for the qubits to assume simultaneously. 2^{300} is actually greater than the number of particles in the universe! So if every particle in the universe were used to do a computation on a classical computer, it would not be enough to simulate the dynamics of a 300 qubit system.

It must be noted, however, that when a quantum system is measured, it will collapse to only one of the potential states. For example, in the 2 qubit case, when $|\psi\rangle$ is measured, it will collapse to one of either $|00\rangle$, $|01\rangle$, $|10\rangle$ or $|11\rangle$ with probabilities $|\alpha|^2$, $|\beta|^2$, $|\gamma|^2$ and $|\delta|^2$ respectively. While it may seem that nothing has been gained by using the quantum bit, it turns out that it is possible to profit from this large encoding by designing computations in such a way that these superpositions interfere. This interference allows an otherwise probabilistic measurement to become deterministic. We refer to these operations that allow us to manipulate qubits as quantum gates.

It might be appropriate to ask, then, what kinds of computations are tailored in this way? Although there is little gain in most tasks that can be done on a quantum computer, it has been shown that there exist quantum algorithms that can be performed exponentially faster than any deterministic classical algorithm. In 1992, David Deutsch and Richard Jozsa proposed the first quantum algorithm to have this property [12]. Although it was quite simple, it had significant implications for the field of quantum information and gave rise to some very important quantum algorithms, most notably Shors algorithm and Grovers search algorithm.

In 1994 Peter Shor discovered an algorithm that allows a quantum computer to factor large numbers exponentially faster than any classical algorithm. This was a significant catalyst for the quantum information movement and became a major interest in the study of cryptography due to its potential to crack RSA keys. Grovers search algorithm is a quantum procedure used to achieve amplitude boosting and was shown to have a polynomial speed-up compared to its classical counterpart. Grover's search algorithm is particularly interesting because it allows one to search through an unsorted database of O(n) in $O(\sqrt{n})$ steps. Grover's search algorithm is extremely versatile, thus it is regarded as one of the most important quantum algorithms to date.

2.3 Universal Quantum Computation

In general, one seeks a protocol for universal quantum computation, that is, an architecture that captures the full power of quantum computation. The conditions required to implement a protocol for universal quantum computation have been stated by David DiVincenzo [13] as:

state initialization of the qubits

long-lived coherences

universal set quantum gates

efficient qubit measurement

scalable to large numbers of qubits

Most of these requirements have been demonstrated in various trapped ion systems. State initialization in trapped ion systems has been demonstrated through optical pumping to a well known hyperfine state. Ions have also been demonstrated to have very long coherence times, and in fact have been shown to have coherence times greatly exceeding the average duration of many quantum gates [14]. A universal set of quantum gates is a set of gates that allow any possible operation to be achieved on a quantum computer. Essentially, any quantum evolution or unitary operation can be expressed in terms of a finite sequence of the universal quantum gates. An example of a universal set of quantum gates for a 2 qubit system are an arbitrary rotation and a set of entangling gates. Efficient qubit measurement is necessary for the success of any operation performed on a qubit and scalabity is required to perform bigger tasks.

This thesis focuses on the requirements of state initialization of the qubits and the efficient qubit measurement. Since quantum computers are highly susceptible to noise, they require not only that a qubit be initialized and measured with high accuracy, but also be robust enough so that quantum gates may be implemented. To successfully achieve fault-tolerant quantum computation, a lower limit must be placed on the fidelities for state initialization, logic gates, and measurement. More specifically, the measurement fidelity must be high enough such that the output of the quantum computer is meaningful and any necessary quantum error correction scheme can be implemented. In general, maximizing the qubit readout fidelity is highly important in cluster state quantum computation. Moreover, this maximization allows one to compensate for logic gate errors in correction schemes. High fidelity state detection is also particularly crucial is tomographic density matrix reconstruction. [15].

In the case of trapped ion quantum computation, a qubit is generally created out of a hyperfine structure in the ground state of the ion or some optical transition between a ground state and a metastable state. This thesis will focus on the a hyperfine clock qubit in Ytterbium. State preparation is achieved through optical pumping and application of microwaves at the qubit splitting, and state detection is achieved through the collection of photons resulting from a cyclical excitation transition. This will be discussed in further detail in chapter 3.

CHAPTER III

Ion Trapping and Ytterbium

"I hate these filthy Neutrals, Kif! With enemies you know where they stand, but with Neutrals? Who knows! It sickens me."

-Zapp Brannigan (Futurama)

Trapped atomic ions have long been considered ideal candidates for the implementation of quantum computation and quantum information processing [10, 16]. This is primarily due to long trapping lifetimes, long coherence times and exquisite control of both internal and external degrees of freedom. The ion used in this experiment is Ytterbium, which has hydrogen-like structure with a spin 1/2 nucleus. Neutral Ytterbium has two valence electrons, so ionized Ytterbium will therefore have one valence electron and essentially act as a heavy hydrogen atom. Ytterbium is a particularly strong candidate for this implementation for a few reasons. First, Ytterbium has a strong coupling between the ${}^{2}S_{1/2}$ and ${}^{2}P_{1/2}$ levels at 369.5261 nm, which is suitable for optical fibers, thus allowing implementations that require coupling of atomic to photonic qubits to be quite accessible [17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31]. Furthermore, Ytterbium has a large fine structure splitting which allows fast manipulation with broadband laser pulses [32, 33, 34, 35]. In table 3.1, various atomic properties of Ytterbium are shown.

3.1 Ion Traps

In the experiment described in chapter V, we utilize a radiofrequency (rf) ion trap. The rf ion trap was invented by Wolfgang Paul, and it earned Paul the Nobel Prize in 1989. The ion trap is an incredibly versatile tool in atomic physics, as it has been used in mass spectroscopy, atomic clocks, measurements of fundamental constants and quantum information processing. We now do a brief overview of ion trap basics,

	Yb ⁺
isotope (amu)	171, 173
nuclear spin	1/2, 5/2
$^2S_{1/2}$ hfs (GHz)	12.6, 10.5
P fs (THz)	100
$^{2}S_{1/2}\leftrightarrow ^{2}P_{1/2}$	369.5
${}^2S_{1/2} \leftrightarrow {}^2P_{3/2}$	329
$^{2}D_{3/2}\leftrightarrow ^{2}P_{1/2}$	2438
$^{2}D_{3/2} \leftrightarrow ^{2}P_{3/2}$	1350
$^{2}D_{3/2} \leftrightarrow ^{3}[3/2]_{1/2}$	935.2
$^{2}D_{5/2} \leftrightarrow ^{2}P_{3/2}$	1650

Table 3.1: Ytterbium Parameters. The transition numbers are in nm. Many of the numbers quoted here are taken from the NIST database [36].

including a discussion of the equations of motion as well as a discussion of the Paul trap variant that is used in the experiment.

3.1.1 Ion Trap Basics In order to trap a charged particle in 3 dimensions, one must use a combination of static and dynamic electric fields or a combination of electric and magnetic fields. In this experiment, we use an rf Paul trap, which uses static and time varying electric fields. The reason that a charged particle cannot be confined by static fields is a result of Gauss' Law. Recall that in free space, $\vec{\nabla} \cdot \vec{E} = 0$. Notice that the electric field in free space behaves like an incompressible fluid in that for any particular Gaussian enclosure, any electric field lines that enter that enclosure must also exit. Therefore, there is no configuration of static electric fields that will be trapping in all directions. The idealized rf Paul trap consists of two hyperbolic electrodes as endcaps that use static fields to trap the ion in the \hat{z} direction and a ring electrode that confines the ion in the x-y plane with oscillating fields.

The electric potential must satisfy Laplace's Equation, that is $\Delta \Phi = 0$. Thus, in order to extract a quadrupole potential that satisfies this in 3 dimensions, the potential must take the form

$$\Phi = \frac{\Phi_0(r^2 - 2z^2)}{r_0^2 + 2z_0^2} \tag{3.1}$$

which is a solution in cylindrical coordinates.

Here, Φ_0 is the applied voltage onto the electrode, and so that may be written in terms of a DC term and an AC term. The resulting equation is

$$\Phi = \frac{(U + V\cos(\omega t))(r^2 - 2z^2)}{r_0^2 + 2z_0^2}$$
(3.2)

From here, one may easily generate the equations of motion for the ion.

$$\ddot{r} + \frac{2e}{m(r_0^2 + 2z_0^2)} (U + V\cos(\omega t))r = 0$$
(3.3)

$$\ddot{z} - \frac{4e}{m(r_0^2 + 2z_0^2)} (U + V\cos(\omega t))z = 0$$
(3.4)

Equation 3.4 can be recognized as the form of the Mathieu equations, which take the form

$$\frac{d^2r}{d\tau^2} + (a + 2q\cos(2\tau))r = 0 \tag{3.5}$$

$$\frac{d^2z}{d\tau^2} - (a + 2q\cos(2\tau))z = 0 \tag{3.6}$$

Simple manipulation of the constants in 3.3 and 3.4 allows us to recover 3.5 and 3.6. The solutions to the Mathieu equations are well known, and stable solutions of the Mathieu equations depend on the constants a and q. Solving the Mathieu equations and using a low order quantum approximation shows that the ion will exhibit simple harmonic motion in the \hat{r} and \hat{z} directions that is modulated with another oscillatory motion known as micromotion. This micromotion cannot be removed from the system and it oscillates at the rf drive frequency of the trap [37].

3.1.2 The Four Rod Trap In general, it is tedious and thus not very efficient to use the hyperbolic electrodes described above. In practice, the structure of the ion trap needs to be altered to allow for optical access, trapping zones, and other experimental constraints. Regardless, ion traps of different geometries can be constructed such that an ion will obey equation 3.4. In the experiment described below, we use a linear rf Paul trap with 4 rods and two needle endcaps. The 4 rods are .5 mm in diameter and the rods are spaced equally by .5 mm. The two endcaps are separated by 2.6 mm. We apply an rf voltage to two diagonally opposite rods and keep the other two grounded while applying a DC voltage to the two needles.



Figure 3.1: This is a schematic of the four rod trap used in the experiment. Schematic courtesty of Steve Olmschenk

The rf drive frequency of the trap is 37 MHz. DC voltages are also applied to the 4 rods in order to reduce residual micromotion caused by imperfect positioning of the ion in the trap [38]. A schematic of the trap used in the experiment can be found in figure 3.1.

3.2 Photoionization

The trapping process begins by a thermal spray of Ytterbium atoms through resistively heating an Yb source oven. A continuous-wave (cw) diode laser tuned to 398.91 nm, which corresponds to the $S_0 \leftrightarrow P_0$ transition in neutral Yb, is focused to the center of the trap with a beam waist of roughly 50 μ m. A second beam at approximately 369.53 nm is aligned counter propagating to the first beam. This second beam is generated by frequency doubling a cw diode laser near 739.05 nm. Together, these two beams photoionize the Ytterbium atoms through a dichroic two-photon transition [39, 8].

3.3 Doppler Cooling

After an Yb ion has been trapped, the atom is Doppler cooled using the laser at 369.53 nm, which is slightly red detuned from the ${}^{2}S_{1/2} \leftrightarrow {}^{2}P_{1/2}$ transition. Figure 3.3 shows this transition along with Ytterbium structure and relevant transitions required to achieve cooling, state initialization and detection. Doppler cooling is dependent on the frequency detuning relation to the photon scattering rate of the ion, given by equation 3.7 [40].

$$\Gamma_{scattering} = \frac{\Gamma}{2} \frac{s_0}{1 + s_0 + 4(\frac{\Delta}{\Gamma})^2}$$
(3.7)

Here, $s_0 = I/I_{sat}$ is the saturation parameter, Γ is the spontaneous emission rate given by the natural lifetime of the excited state, and Δ is the detuning of the incident beam from resonance (in Hz). Since the laser is red-detuned, when the ion moves toward the beam, it will scatter more photons than when it is moving away from the beam, resulting in a net momentum kick toward the trap center. This, combined with the restoring force of the ion trap, will allow the ion to be cooled. Moreover, the laser is aligned so that it is not perpendicular to any principal axis of the trap, allowing a single beam to cool the atom in every direction.

In this experiment, it is not enough to cool the ion solely with the 369.53 nm light. To prevent population trapping in the ${}^{2}S_{1/2}|F = 0$ state caused by off-resonant coupling to the ${}^{2}P_{1/2}|F = 1$ manifold, the 369.53 nm light is sent through a bulk resonant electro-optic modulator (EOM) that is driven at 7.37 GHZ. The positive second order sideband is resonant with the ${}^{2}S_{1/2}|F = 0$ $\leftrightarrow {}^{2}P_{1/2}|F = 1$ transition, which allows the ion to return to the cooling cycle. Because the ${}^{2}P_{1/2}$ state also decays to the metastable ${}^{2}D_{3/2}$ state with a measured probability of .005 (this is derived from the branching ratio), light at 935.2 nm is used to drive the atom to the ${}^{3}[3/2]_{1/2}$ state, from which it rapidly decays back to the ${}^{2}S_{1/2}$ ground state [41]. Unfortunately, another issue arises due to the low-lying ${}^{2}F_{7/2}$ state, which the ion falls into a few times per hour (likely due to collisions with background gas) [42, 43]. Using 638.6 nm light, the ion is returned to the regular cooling scheme.

3.4 The Hyperfine Qubit

In Ytterbium, the qubit is stored in the hyperfine states of ${}^{2}S_{1/2}$, namely $|F = 1, m_{F} = 0\rangle$ and $|F = 0, m_{F} = 0\rangle$. In order to define a quantization axis and avoid a coherent superposition of the three ${}^{2}S_{1/2}|F = 1\rangle$ states, which is not coupled with the detection beam, we apply a magnetic field of roughly 3.4 Gauss to to the trap. This is important because the resulting hyperfine qubit is insensitive to magnetic fields to first order [44]. We define ${}^{2}S_{1/2}|F = 1, m_{F} = 0\rangle$ as the logical state $|1\rangle$, and the ${}^{2}S_{1/2}|F = 0, m_{F} = 0\rangle$ state as the logical state $|0\rangle$. F is defined as the total angular momentum of the ion, I + J, and m_{F} is its projection along the quantization axis. The hyperfine splitting of these two states is 12.642812118466 + δ GHz where δ is the second order Zeeman shift. This second order Zeeman shift can be calculated by



Figure 3.2: Ytterbium 171 Energy Diagram. This shows all energy transitions used in the experiment. Bold lines represent transitions driven by laser sources. The numbers in parentheses are branching ratios and several lifetimes are provided as well. Courtesy of Steve Olmschenk

diagonalizing the Hamiltonian for the ion in the presence of an external magnetic field, given by 3.8.

$$\hat{H} = \alpha \vec{I} \cdot \vec{J} + (g_J \mu_B m_J - g_I \mu_B m_I) B_z \tag{3.8}$$

Here, α is the hyperfine coupling term, \vec{I} is the spin of the nucleus, \vec{J} is the sum of the orbital angular momentum (\vec{L}) and the spin (\vec{S}) of the electron, g is the Lande g factor, m is the z component of the labeled angular momenta, B_z is the z component of the magnetic field, and μ_B is the Bohr magneton. By finding the eigenvalues of this Hamiltonian, one can derive the second order Zeeman shift.

3.5 State Initialization

State initialization is achieved through optical pumping into $|0\rangle$, as shown in figure 3.3. Before being sent to the ion, the 369.53 nm beam is passed through a bulk-resonant EOM driven at 2.1 GHz (which is the hyperfine splitting of the ${}^{2}P_{1/2}$ manifold). The resulting positive first order sideband is resonant with the ${}^{2}S_{1/2}|F = 1\rangle \longleftrightarrow {}^{2}P_{1/2}|F = 1\rangle$ transition. From the ${}^{2}P_{1/2}|F = 1\rangle$ manifold, there is a 1/3 chance that the ion will fall into the $|0\rangle$ state. However, since the ${}^{2}P_{1/2}|F = 1\rangle$ state also decays to ${}^{2}D_{3/2}$, 935 nm light is sent through a fiber EOM driven at 3.07 GHz and the positive first order sideband is used to pump out of that state into the initialization cycle. Using approximately 8 μ W of 369.53 nm light with a waist of about 30 μ m at the trap, the ion is optically pumped into $|0\rangle$ in less than 500 ns with near perfect efficiency. To initialize the ion into $|1\rangle$, a microwave π pulse can be applied at the qubit hyperfine frequency [8].

3.6 State Detection

State detection is critical to the success and a crucial step in quantum information protocols. State detection is achieved using a fluorescence method by exploiting the ${}^{2}S_{1/2}|F = 1\rangle \leftrightarrow {}^{2}P_{1/2}|F = 0\rangle$ transition [16, 45, 15]. The detection light will therefore be tuned nearly to resonance at 369.53 nm. If the ion is initialized in $|0\rangle$, then the incident light is detuned from ${}^{2}P_{1/2}|F = 0\rangle$ by 14.7 GHz (the sum of the hyperfine frequencies of the ground and excited states). As a result, few photons are scattered in this process, and the ion is "dark." The logical qubit state $|0\rangle$ is thus commonly referred to as the dark state. If the ion is initialized in $|1\rangle$, then ion will scatter many photons and appear bright since the incident light is essentially on resonance. The logic qubit state $|1\rangle$ consequently is referred to as the bright state. The light scattered from the ion goes through a collection lens with a numerical aperture of .27 and is collected by a photomultiplier tube (PMT) with quantum efficiency .35.

There are generally two schemes that exploit these properties to detect the state of the ion: a discriminator method, which counts the number of photons collected by the PMT during the detection interval and compares the total amount to a threshold value, and a method that employs the method of maximum likelihood and conditional probabilities based on photon arrival times. The following two schemes will be explained in greater detail in chapter 4.

CHAPTER IV

Methods of State Detection

"You can know the name of a bird in all the languages of the world, but when you're finished, you'll know absolutely nothing whatever about the bird... So let's look at the bird and see what it's doing – that's what counts."

-Richard P. Feynman

State detection in trapped ion systems is accomplished using standard fluorescence techniques. In the case of Ytterbium, the ion is hit with a beam nearly on resonance with the ${}^{2}S_{1/2}|F = 1\rangle \leftrightarrow {}^{2}P_{1/2}|F = 0\rangle$ transition and the ion fluoresces with a rate given by its current state, either $|0\rangle$ or $|1\rangle$. The scattered photons are then observed by a PMT with a quantum efficiency of .35. If the ion is in the bright state, it will scatter photons at a rate of roughly 12500 photons per second. If the ion is in the dark state, it will scatter virtually 0 photons.

In the following sections there will be a discussion on two significant detection schemes and the errors that limit the fidelity of qubit state detection.

4.1 Discriminator Method

State detection using the discriminator method exploits the fact that an ion will scatter many photons in the bright state and virtually none in the dark state. Using an Field Programmable Gate Array (FPGA) circuit, we are able to count photons that are observed by the PMT during the detection period. In an ideal experiment, an ion in the bright state will emit photons with Poissonian statistics and the mean number of photons detected will be given by the length of the detection interval. Theoretically, if one were to detect for an infinite amount of time, then the discriminator error would converge to zero. Of course, since state leakage exists and it is not feasible to run an infinite amount of experiments, errors must be considered. An ion in the dark state will generally register 0 photon counts, but due to background scattering and state leakage, there are a significant number of non-zero counts registered. In this method, if more than 1 photon is observed during the detection interval, then the ion is inferred to be in the bright state; if one or zero photons are detected, the ion is inferred to be dark. A histogram showing photon counts normalized to show probability for both the bright and dark states can be found in figure 5.1.

There are several sources of error in this method. The limiting source of error derives from off-resonant coupling to the ${}^{2}P_{1/2}|F = 1\rangle$ manifold. Given a photon collection efficiency of .1%, the maximum detection fidelity for 171 Yb is 99.51%. Off resonant coupling to the ${}^{2}P_{1/2}|F = 1\rangle$ level when the ion is initialized in $|1\rangle$ accounts for much of this error. This coupling occurs because the hyperfine splitting of the ${}^{2}P_{1/2}$ level is only 2.1 GHz. If the initial state is $|0\rangle$, then the beam is detuned from the ${}^{2}P_{1/2}|F = 1\rangle$ level by 14.7 GHz, so transitions to the ${}^{2}P_{1/2}|F = 0\rangle$ level are impossible due to selection rules. Additionally, decay to the metastable ${}^{2}D_{3/2}$ state will reduce the total number of photons seen by the PMT.

A significant issue is also caused by coherent population trapping in the ${}^{2}S_{1/2}|F = 1\rangle$ manifold. Coherent dark states are prevented by breaking the degeneracy of the F = 1 manifold with a magnetic field. The resulting Zeeman shift results in a detuning for the laser which reduces the scatter rate by roughly a factor of 3. This leads to an overall theoretical detection fidelity of 98.55%.

4.2 Photon Arrival Times

In the previous method, an ion was projected into either the bright or dark state based on how the number of photons collected compared to a discriminator value. While the discriminator method has proven to be effective, there is a more efficient method based on the principle of maximum likelihood using photon arrival times. Essentially, this method evaluates the likelihood that a given set of photon arrival times will be generated given that the ion is initialized in a certain state. It seems logical that there is more information in the photon arrival times. We will follow the basic method described by Langer [1].

We are presented with the problem of inferring whether the ion is dark $(|0\rangle)$ or bright $(|1\rangle)$ given the arrival times of N photons, $\{t_i | i \in \{1, 2, ..., N\}\}$ during a detection interval τ_D . To do this, we must introduce the likelihood function, $L[\{t_i\} | |\psi\rangle]$. Mathematically, this represents a conditional probability of generating the set $\{t_i\}$ given the ion is in state $|\psi\rangle$, where $\psi \in \{0, 1\}$.

To construct the likelihood function, we must consider all possible ways in which each photon of the set was emitted. The recipe for constructing the likelihood function follows the Law of Total Probability and a special form of Bayes' Theorem. First, Consider a single event, X. Given that only distributions A and B can exclusively generate this event, we can write the total probability of the event to occur in the following way:

$$P(X) = P(X|A)P(\bar{X}|B) + P(\bar{X}|A)P(X|B)$$

$$(4.1)$$

Here, \bar{X} is defined as the complement of X. In order to define the probability that a single event X will occur, we must sum over the probability that distribution A generated X and distribution B did not AND the probability that distribution B generated X and A did not. This is an essential rule that must be used to derive the likelihood function. In the case of state detection, we can define X as the event of a single photon being detected by the PMT and \bar{X} as being the event that zero photons are detected by the PMT.

4.3 Distributions of Photon Emission

In the previous section, we described the probability to generate a single photon. This probability is conditioned by the distributions of photon emission. In the follow two subsections, we describe the theoretical distributions in the ideal case and in the case of our experiment.

4.3.1 Theoretical Distributions We begin by looking at the distribution of photon emission by the dark state. Clearly, the dark state on its own will scatter virtually zero photons. However, since the dark state can leak to the bright state, the distribution of photons will not be a perfect delta function at n = 0 photons. The dark to bright state leakage event can be described by an exponential probability density function with a decay parameter $\gamma_{fb/fd}$ where fb refers to the bright parameter and fd refers to the dark parameter.

$$f(t)dt = \gamma_{fb/fd} e^{-\gamma_{fb/fd}t} dt \tag{4.2}$$

Naturally, if one considers the probability that the ion has decayed in some arbitrary time interval (t, t + T) of length T, then one must consider the cumulative distribution function of the flipping event. It is important to note that two flipping events in different time intervals are independent, thus we can always write the cumulative distribution function as follows

$$F(t) = \int_0^T \gamma_{fb/fd} e^{-\gamma_{fb/fd}t} dt$$
(4.3)

When the ion is in the bright state, it will emit photons according to Poissonian statistics. Recall that the Poisson distribution has the following form:

$$P(N(t+T) - N(t) = k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

$$\tag{4.4}$$

Here, N(t + T) - N(t) denotes the number of photons in an interval (t, t + T] and λ denotes the mean number of photons expected in the interval. We can express λ as a function of the time t, when the ion flipped from dark to bright by

$$\lambda(t) = (1 - \frac{t}{\tau_D})\lambda_0 \tag{4.5}$$

The parameter λ_0 is the mean number of photons expected in detection interval τ_D when initialized in the bright state. We wish to turn the original probability density function for the flipping event into a Poissonian, so will solve 4.5 to get t as function of λ and insert this relation into 4.2. The resulting equation dictates the probability that a dark ion will emit photons according to Poissonian statistics with expected mean λ and total collection efficiency η :

$$P_D(\lambda) = \begin{cases} e^{\frac{-\lambda_0 \alpha_1}{\eta}} & : \lambda = 0\\ \frac{\alpha_1}{\eta} e^{\frac{(\lambda - \lambda_0) \alpha_1}{\eta}} & : \lambda > 0 \end{cases}$$

The parameter α_1 describes the leak probability per emitted photon and this can be derived theoretically using relevant atomic parameters of Ytterbium. The discontinuity at $\lambda = 0$ accounts for the case that the ion never leaves the dark state. In order to find the ultimate dark state distribution, this dark state Poissonian must then be convolved with a bright state Poisson distribution [15]:

$$D(n) = e^{\frac{-\lambda_0 \alpha_1}{\eta}} \delta_n + \int_x^{\lambda_0} \frac{\alpha_1}{\eta} e^{\frac{(\lambda - \lambda_0) \alpha_1}{\eta}} \frac{\lambda^n e^{-\lambda}}{n!} d\lambda$$
(4.6)

Here, δ_n is the Kronecker delta function and this integral is done in the limit that $x \to 0$. Notice that in this limit, the integral converges to the lower incomplete gamma function. The normalized lower incomplete gamma function is defined as $\Gamma_{lower}(x, a) = \frac{1}{(a-1)!} \int_0^x e^{-z} z^{a-1} dz$. Furthermore, this can be written in terms of the gamma function such that $\Gamma_{lower} = \frac{1}{(a-1)!} (\Gamma(a) - \Gamma_{upper}(a, z))$. Thus, rewriting our equation for the dark distribution, we find

$$D(n) = e^{\frac{-\lambda_0 \alpha_1}{\eta}} \delta_n + \frac{\alpha_1}{(1-\alpha_1)^{n+1}} \Gamma_{Lower}(n+1, (1-\alpha_1)\lambda_0)$$
(4.7)

A plot of this distribution with a measured value of $\alpha_1 = .0006$ can be found in figure 4.1a. Using a similar procedure, one can construct the bright state distribution.

$$B(n) = \frac{e^{-(1+\alpha_2/\eta)\lambda_0}\lambda_0^n}{n!} + \frac{\alpha_2/\eta}{(1+\alpha_2/\eta)^{n+1}}\Gamma_{Lower}(n+1,(1+\alpha_2/\eta)\lambda_0)$$
(4.8)

Here, α_2 represents the bright to dark leakage parameter and is also determined by theoretical means. A plot of this distribution with a measured value of $\alpha_2 = .0076$ can be found in figure 4.1b.

4.3.2 Theoretical Distributions With Background In the previous subsection, the theoretical distributions of photon emission were derived for the bright state and dark state assuming that there was no contribution from background scattering. While the bright state is modeled quite well, the dark state distribution is quite different from what we actually see experimentally. We now revisit the analysis provided by Acton et al. with a full treatment of the background distribution.

We begin our analysis by assuming that the background scattering follows Poisson statistics with a mean number of counts λ_{bg} . If the dark state leaks to the bright state at some time t, then the collected photons will exhibit Poisson statistics with mean given by

$$\lambda(t) = \lambda_{bg} + \gamma_b(\tau_D - t) = \lambda_{bg} + \lambda_0 - \lambda \tag{4.9}$$

Here, γ_b refers to the bright state scattering rate and λ_0 refers to the mean number of photons expected from the bright state in a detection time of τ_D . Now, recall that the probability density function for the flipping event is described by an exponential probability density function:

$$f(t)dt = \gamma_f e^{-\gamma_f t} dt \tag{4.10}$$

By solving for the flipping time t, and turning the flipping equation into a Poissonian, we can get the probability for the dark ion to exhibit Poissonian statistics with mean λ . This is given by the following.

$$g(\lambda) = \begin{cases} \frac{\gamma_f}{\gamma_b} e^{-\frac{\gamma_f}{\gamma_b}(\lambda_{bg} + \lambda_0 - \lambda)} & : \lambda_{bg} < \lambda \le \lambda_{bg} + \lambda_0 \\ \\ e^{-\gamma_f \tau_D} & : \lambda = \lambda_{bg} \end{cases}$$



Figure 4.1: Theoretical Distributions. a) Theoretical distribution described by 4.7. The smaller plot in the top right corner is a zoomed out view of the distribution. b) Theoretical distribution described by 4.8



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Figure 4.2: The dark state distribution

Now, following Acton's method of deriving the dark state histogram, we convolve $g(\lambda)d\lambda$ with a Poissonian of mean λ , that is, $\frac{\lambda^n e^{-\lambda}}{n!}$.

$$D(n) = e^{-\gamma_f \tau_D} \left(P(n|\lambda_{bg}) + \frac{\gamma_f e^{-\gamma_f \lambda_{bg}/\gamma_b}}{\gamma_b n!} \int_{\lambda_{bg}}^{\lambda_{bg}+\lambda_0} e^{\gamma_f \lambda/\gamma b} e^{-\lambda} \lambda^n d\lambda \right)$$
(4.11)

Using the lower incomplete gamma function and some simplification we are left with the following relation:

$$D(n) = e^{-\gamma_f \tau_D} \left\{ \frac{\lambda_{bg}^n e^{-\lambda_{bg}}}{n!} + \frac{\gamma_f \gamma_b^n e^{-\gamma_f \lambda_{bg}/\gamma_b}}{(\gamma_b - \gamma_f)^{n+1}} \left[\Gamma_l \left(n + 1, (\gamma_b - \gamma_f)(1 + \frac{\lambda_{bg}}{\lambda_0 \tau_D}) \tau_D \right) - \Gamma_l \left(n + 1, (\gamma_b - \gamma_f) \frac{\lambda_{bg}}{\lambda_0} \right) \right] \right\}$$

$$(4.12)$$

A plot of this distribution is in figure ??.

Immediately, we can test the discriminator method theoretically. The theoretical fidelity for the dark state is 99.59%, which coincides with what we see.



??

Figure 4.3: The bright state distribution

Now we use the same procedure to get the bright state histogram except now we essentially have a swap between λ_0 and λ_{bg} . Let λ_B be equal to the sum $\lambda_0 + \lambda_{bg}$. After simplification we find the following for the bright state distribution:

$$B(n) = e^{-\gamma_f \tau_D} \left\{ \frac{\lambda_B^n e^{-\lambda_B}}{n!} + \frac{\gamma_f \gamma_b^n e^{\gamma_f \tau_D \lambda_{bg}/\lambda_0}}{(\gamma_b + \gamma_f)^{n+1}} \left[\Gamma_l \left(n + 1, (\gamma_b + \gamma_f)(1 + \frac{\lambda_{bg}}{\lambda_0})\tau_D \right) - \Gamma_l \left(n + 1, (\gamma_b + \gamma_f) \frac{\lambda_{bg}}{\lambda_0}\tau_D \right) \right] \right\}$$

$$(4.13)$$

A plot of the bright state distribution can be found in figure ??.

4.4 The Likelihood Function

To derive the likelihood function, we must use the principle described by 4.1 to write out all possible scenarios in which each initial state produced the photon arrival list. Consider a detection time τ_D of which we break up into N sub-bins of length T. Photons are collected by the PMT and this information is stored for each time bin T of the detection interval. The probability that n photons are detected in each time interval will be summed up over the entire detection time.

The probability that n photons are detected in a given time bin follows the law of Total Probability. For example, given that the ion is initialized in the dark state, the probability a photon is detected in the first time bin can be described by two possible scenarios: the photon was generated by the dark distribution or the ion flipped states and the photon generated by the bright distribution. After summing over all possibilities, we derive the final likelihood function.

For the dark state, the likelihood function is given by

$$L_d = (e^{-\gamma_{fd}\tau_D}) \prod_{i=1}^N D(n_i) + (1 - e^{-\gamma_{fd}T}) \sum_{j=1}^N \prod_{k=1}^{j-1} D(n_k) \prod_{\ell=j}^N B(n_\ell)$$
(4.14)

The $e^{-\gamma_{fd}\tau_D}$ term represents the probability that the ion never flipped in the detection interval and the term $(1 - e^{-\gamma_{fd}T})$ represents the probability to flip from dark to bright in any given time bin.

Since $\gamma_{fd}T$, $\gamma_{fd}\tau_D \ll 1$, we can rewrite the likelihood function to a good approximation [2] as

$$L_d = (1 - \gamma_{fd} \cdot \tau_D) \prod_{i=1}^N D(n_i) + \gamma_{fd} T \sum_{j=1}^N \prod_{k=1}^{j-1} D(n_k) \prod_{\ell=j}^N B(n_\ell)$$
(4.15)

Using the same principles, we can construct the likelihood function given the ion is initialized in the bright state, and approximate it in the same fashion:

$$L_b = (e^{-\gamma_{fb}\tau_D}) \prod_{i=1}^N B(n_i) + (1 - e^{-\gamma_{fb}T}) \sum_{j=1}^N \prod_{k=1}^{j-1} B(n_k) \prod_{\ell=j}^N D(n_\ell)$$
(4.16)

$$L_b = (1 - \gamma_{fb} \cdot \tau_D) \prod_{i=1}^N B(n_i) + \gamma_{fb} T \sum_{j=1}^N \prod_{k=1}^{j-1} B(n_k) \prod_{\ell=j}^N D(n_\ell)$$
(4.17)

To determine whether a given set of photon arrival times has been generated by the dark state or the bright state, we introduce the likelihood ratio, $L_r \equiv \frac{L_b}{L_d}$. If L_r is greater than 1, the ion is projected into the bright state. Conversely, if it is less than 1, the ion is projected into the dark state. In the highly unlikely event that the likelihood ratio is equal to 1, no inference can be made about the state of the ion. Using this likelihood ratio to infer the state of the ion accounts for several limiting errors found in the discriminator method.

By comparing the two likelihood ratios, we can account for a few major sources of error. First and foremost, in the event that only one photon is detected early in the detection interval, the likelihood ratio will recognize this as an initial bright ion that emitted a photon and pumped to the dark state. This occurs quite often experimentally and the discriminator will always fail to detect this. Furthermore, by considering a long string of photons at the end of the detection interval, the likelihood function compares the flipping functions of both the bright and dark state and recognizes that it is more likely the ion was dark and pumped to the bright state. Again, the discriminator method will falsely project this into the bright state. In addition to these benefits, if multiple photons are emitted, the likelihood function of the bright state will be slightly increased, because the density function for the bright state would predict more photon emission. This implicity utilizes the discriminator method while taking into account their respective time signatures.

4.5 First Photon Method

The likelihood method above implicitly places a lot of significance on the arrival time of the first photon [1]. We noted in section 4.1 that the discriminator method is particularly effective in detecting the the dark state. Since we are limited by the bright state due to state leakage, the discriminator method is far less effective in determining the bright state. The error by the discriminator method is a direct result of the overlap between the bright state and dark state histograms, which primarily occurs for the case that only one photon is emitted. When the bright state leaks to the dark state, it is fairly common to see one photon emitted followed by darkness, which the discriminator infers as dark. We now present an alternative method to the likelihood method and discriminator method which combines them in the following way.

Considering that the one photon case is what limits the discriminator method for the bright state, we will now alter the discriminator method when a single photon is detected. In theory, it is much more likely that if a single photon is observed early in the detection interval, that the ion is bright. Therefore, for the single photon case, we will define a threshold time t_c that determines the state of the ion. If the single photon arrive before t_c , then the ion is inferred to be bright. Likewise, if the single photon arrives after t_c , then the ion is inferred to be dark. For all other cases, the usual discriminator method is used.

CHAPTER V

Simulations

"The truth is a beautiful and terrible thing, and should therefore be treated with caution."

-Albus Dumbledore (Harry Potter)

In the previous chapter, two main detection schemes employed in ion trapping qubit readout were described. In this chapter, we describe simulations used to test the validity of the maximum likelihood method and to verify that the likelihood function theoretically is more efficient than the discriminator. We first will outline the procedure used to generate random photon arrivals from both the dark and bright states. Then, we will highlight the procedure used to calculate the theoretical error of both the discriminator method as well as the likelihood method.

5.1 Simulating Random Photons

In order to test the validity of our maximum likelihood method of state detection, we must find a way to generate random photons. We begin by defining a detection interval τ_D and the number of sub-intervals of this detection time, *Timebins*. Essentially, a probability density function is generated by comparing a random number between 0 and 1 with the probability to flip states and the probability to generate a single photon. For each time bin, the probability to flip is evaluated before the probability to generate a single photon. After determining whether the ion has flipped or not, a random number is compared to the appropriate probability for the ion to emit a photon.

The probability to flip within a given timebin is given by the cumulative density function of the flip function defined in chapter 4. The probability to generate a single photon, however, differs significantly from the theory introduced previously. We must now introduce a joint probability density function that describes the photon emission from the background and the bright state. It is expected that in the ideal case that both the background distribution and a bright ion will emit photons according to Poissonian statistics. Using this as a basis for the probability density functions of both the background and bright ion, one can construct the joint probability density function.

Recalling the Poisson distribution, if one wishes to evaluate the probability that a single photon is generated, then the Poisson distribution converges to an exponential distribution with parameter λ . Thus, we can construct the bright state and background distributions as two separate exponential distributions:

$$P_b(t)dt = \gamma_b e^{-\gamma_b t} dt \tag{5.1}$$

$$P_{bg}(t)dt = \gamma_{bg}e^{-\gamma_{bg}t}dt \tag{5.2}$$

To create a joint probability density function for the bright and background, we recognize that the joint probability density function of two independent, identically distributed random variables with exponential probability density functions is also an exponential probability density function with a parameter of the sum of the two individual parameters. Therefore, the joint probability density function is

$$P_{b+bg}(t)dt = (\gamma_b + \gamma_{bg})e^{-(\gamma_b + \gamma_{bg})t}dt$$
(5.3)

If an ion is in the bright state, we assume that the ion will generate 0 photons and any photons generated will be solely from the background distribution.

An array of length *Timebins* is initialized and for each respective timebin is filled with the number of photons emitted. This array is essentially a list of photon arrival times and can be employed in the maximum likelihood method. To create the density function for an entire detection interval, one must run a large number of experiments and add up all the photons generated in each detection experiment and create a histogram. In figure 5.1 are the simulated bright state and dark state probability density functions for a detection time of 800 μs and 10000 experiments.

These histograms coincide with the experimentally obtained histograms very well. As expected, the bright state histogram is very similar to the theoretical plot shown in. The effect of the background in the dark state histogram is much more transparent, as there are significantly more counts in the 1 photon bin.



Figure 5.1: Simulated histograms for a general detection interval of 800 microseconds. a) Dark state simulated histogram. b) Bright state simulated histogram.

5.2 State Detection Error Simulations

To get a theoretical estimate of the fidelity of the discriminator method along with the likelihood method, it is necessary to run several simulated data points into all the methods and count the number of errors over a large number of experiments. This is done by generating random photon arrivals as described in the previous section and inserting them individually into the likelihood ratio. However, the probability density functions for the likelihood functions must be tailored in a way that they accurately reflect the probability to generate n photons in a single time bin.

In the likelihood functions described by 4.14 and 4.16, it is only necessary to calculate the probability that n photons are generated in a time interval of one time bin. Therefore, to derive the proper probability density function, we set the detection time to be the previous $\tau_D/Timebins$ and then change the number of time bins to 1. thus we define the new probability density functions in the following equations.

$$D(n) = \begin{cases} 0.999367 & : n = 0\\ .000633 & : n = 1 \end{cases}$$
$$B(n) = \begin{cases} 0.904022 & : n = 0\\ .095978 & : n = 1 \end{cases}$$

For the error determination, we ran 50 000 experiments for 50 different detection windows, ranging from 15 microseconds to 850 microseconds. By varying the detection time, it is easy to determine an



Figure 5.2: a) A logarithmic plot of the error versus the detection time for an initial bright ion. b)

A logarithmic plot of the error versus the detection time for an initial dark ion.

optimal value at which the errors for both the discriminator and likelihood methods level out. The error was calculated for both bright state and dark state initialization, and then averaged to find the overall error in state detection.

A logarithmic plot of the bright state detection error and the dark state detection error in figure 5.2 show that the likelihood method was more efficient in correctly projecting the state of the ion. In particular, the likelihood method was more efficient in detecting the bright state of the ion. Conversely, the likelihood method was less efficient than the discriminator method in detecting the dark state of the ion. The maximum bright state fidelity for the likelihood method was 98.71% and the maximum bright state fidelity for the likelihood method was 98.71% and the maximum bright state fidelity for the bright state fidelity for the likelihood method was 98.71% and the maximum bright state fidelity for the bright state fidelity for the likelihood method was 99.71% and the maximum bright state fidelity for the bright state. The asymptotic dark state fidelity for the likelihood method was 99.79%. Averaging the error tables for both led showed that the likelihood method is slightly more efficient for large detection times, with an asymptotic fidelity of 99.10% for the likelihood method and 99.04% for the discriminator method. This is illustrated in figure 5.3



Figure 5.3: A plot of the averaged bright state and dark state errors for both the likelihood method and discriminator method

CHAPTER VI

Experimental Results

"The search for hard-to-vary explanations is the origin of all progress. It's the basic regulating principle of the Enlightenment. So, in science, two false approaches blight progress. One is well known: untestable theories. But the more important one is explanationless theories. Whenever you're told that some existing statistical trend will continue, but you aren't given a hard-tovary account of what causes that trend, you're being told a wizard did it." –David Deutsch

6.1 Experimental Setup

In the following experiment, we will use the hyperfine clock states of the ${}^{2}S_{1/2}$ manifold of an ${}^{171}Yb^{+}$ ion, separated by a frequency of 12.6428 GHz. This ion is trapped in an rf Paul trap with a 1.2 MHz oscillation frequency in the radial direction. The ion is doppler cooled using a 369.53 nm laser that is slightly reddetuned from the ${}^{2}S_{1/2} \leftrightarrow {}^{2}P_{1/2}$ transition. Then the laser is optically pumped to the dark state, $|0\rangle$, where it can be initialized to $|1\rangle$ by applying a microwave π -pulse to the ion. Once the ion is initialized in the desired state, the detection scheme may begin. We send the 369 nm light through an acousto-optic modulator (AOM), then the detect gate is sent to the AOM in the form of a square pulse with a bandwith of 800 microseconds. The AOM will then deflect the beam for 800 microseconds, and optics are set up to send that deflected beam to the ion. It is important to note that during state detection, we send σ^{+} , π , and σ^{-} polarizations of the 369 nm light to the ion in order to avoid coherent dark states. Here, σ^{\pm} refer to the polarizations of the light which correspond to the ${}^{2}S_{1/2}|F = 1$, $m_F = -1\rangle \leftrightarrow {}^{2}P_{1/2}|F = 0$, $m_F = 0\rangle$ and ${}^{2}S_{1/2}|F = 1$, $m_F = 1\rangle \leftrightarrow {}^{2}P_{1/2}|F = 0$, $m_F = 0\rangle$ transitions respectively. Additionally, π polarized light corresponds directly to the ${}^{2}S_{1/2}|F = 1, m_{F} = 0\rangle \leftrightarrow {}^{2}P_{1/2}|F = 0, m_{F} = 0\rangle$ transition. In between detection gates, we Doppler cool the ion and optically pump it to the desired state using the mechanisms described in chapter 3.

Light emitted from the ion focused through a collection lens with a numerical aperture of .27 and then collected by a PMT with quantum efficiency .35. To extract the time signature of each photon collected, the detection gate and PMT signal are sent to an Agilent Mixed Signal Oscilloscope. Using the peak finder setting, one can merely find the time stamp of each spike produced by a photon detected by the PMT for each detection gate. The data was exported to a CSV file and imported into Mathematica for post-processing.

In the post-processing of data, four different state detection methods were used: the likelihood method using the theoretical distributions described by equations 4.7 and 4.8, the the likelihood method using the simulated probability density function given in chapter 5, the first photon method using a t_c of 160 μ s, and the discriminator method.

6.2 Experimental Results

Five thousand detection experiments were run for the case that ion was initialized bright and the ion was initialized dark, which is enough to see a discrepancy between all four detection schemes. The timestamp of each photon was placed into one of the 100 time bins of the detection interval. The four different detection schemes were evaluated for each set of arrival times and if the scheme incorrectly projected the state of the ion, then it was considered an error.

A table of the measured fidelities for each individual method can be found in table 6.1. The error for the bright and dark states was calculated by dividing the total number of incorrect inferences by the total number of experiments. The average error was calculated by taking the arithmetic mean of the errors for the bright and dark states. The fidelity is then $1 - \epsilon$ where ϵ is the average error.

	Bright State Error	Dark State Error	Average Error	Average Fidelity
Likelihood Theory	1.3%	1.5%	1.4%	98.6%
Likelihood Simulated PDFs	1.9%	.2%	1.1%	98.9%
Discriminator	2.1%	.3%	1.2%	98.8%
First Photon	1.5%	.6%	1.1%	98.9%
	Table 6.1: Ex _I	perimental fidelities.		

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Table

The statistical uncertainty for these values can be calculated using a Bernoulli distribution. Using the standard definition of the error, we find that the statistical uncertainty for the case that 5000 experiments were carried out in each state is .02%, so our measured fidelities are a few error bars away from each other, validating the improvement shown by the likelihood and first photon methods.

CHAPTER VII

Conclusion

"Space... It seems to go on and on forever. But then you get to the end and a gorilla starts throwing barrels at you."

-Philip J. Fry (Futurama)

7.1 Summary

In this paper, we have described a rigorous method to detect the state of a trapped ion qubit and tested this theory experimentally using Yb⁺. We counted the number of photons detected by a PMT during a detection interval and timestamped them using a mixed signal oscilloscope, and fed them to the discriminator method and likelihood method. The fidelity for the maximum likelihood method is only slightly better than the discriminator method, giving roughly a .1% increase in fidelity. The likelihood offered a .2% gain in fidelity in detecting the bright state. The discrepancy between the discriminator and the likelihood method is a direct result of the likelihood method projecting the ion into the bright state when only one photon is detected very early in the detection interval. Theoretically, the likelihood method will correctly project the ion into the bright state when the single photon arrives in the first 40 μ s of the typical detection interval of 800 μ m. The likelihood method was approximately .1% worse than the discriminator method in detecting the dark state. Overall, the likelihood method offers essentially no advantage for a single qubit detection scheme. In this final chapter, we investigate the likelihood method's utility in larger systems and future plans.

7.2 Implications in Larger Systems

As noted in chapter 2, two requirements necessary for the realization of universal quantum computation are efficient qubit readout and scalability to a large number of qubits. In the experiment described in this thesis, we considered the qubit readout fidelity for a single qubit. While it is highly important to have efficient qubit readout for a single qubit, the gain provided by the likelihood method for a single qubit is essentially negligible for the single qubit case. However, in order to achieve universal quantum computation, it is necessary to achieve high fidelity qubit readout for a large number of qubits. As more qubits are added to the system, the error increases exponentially. Therefore, the likelihood method gain in fidelity of .1% becomes quite significant in order to have fault-tolerant quantum computation with a large number of qubits.

7.3 Future Plans

While the proposed method of state detection in this thesis does not provide an overwhelming improvement in fidelity for the single photon case, one can implement a few changes to the experiment in order to increase the effectiveness of the maximum likelihood scheme. In the follow sections we outline the future additions and changes that will be implemented in our experiment to increase the fidelity of qubit readout. **7.3.1 The Adaptive Likelihood Function** As shown in the chapter V, the overall error in the likelihood method is only marginally better than the discriminator method. However, the likelihood method can in theory be improved by employing an adaptive version of the maximum likelihood technique [2]. Essentially, the adaptive method allows one to add an estimated correction factor to the likelihood ratio. This allows one to detect the state of the ion with the same minimum error as the likelihood method in an even shorter detection interval. In the previous version of this method, the detection time was fixed and the likelihood functions were calculated at the end of this detection period. If one considers the absolute values of the likelihood functions, more can be derived about the error in state detection. We now introduce the correction factor for both likelihood functions. Given by Bayes' theorem, the estimated error that we have incorrectly inferred the ion to be dark when $L_d > L_b$ is

$$1 - P(|0\rangle | \{t_i\}) = 1 - \frac{P(\{t_i\} | |0\rangle)}{P(\{t_i\})} = \frac{L_b}{L_b + L_d}$$
(7.1)

The correction factor for incorrect inference that the ion is bright can be constructed in a similar

manner. By evaluating the two likelihood functions at the end of each sub-bin of detection interval, one can terminate the detection at some time τ_{cutoff} such that the estimated error probability falls below some threshold error, which is ideally the minimum error of the original two state detection methods. In addition to an overall gain in fidelity, the adaptive likelihood method would allow one to shorten the detection window significantly. Since the adaptive likelihood function is evaluated at the end of each timebin, the a time can be found at which the error in inferring the state of the ion is equal to or better than the likelihood method and discriminator method. Decreasing the gate time while maintaining a high fidelity is particularly important for quantum computational protocols![46].

7.3.2 Additions to the Experiment In addition, the experiment will soon incorporate a new collection lens with a numerical aperture of roughly 3 times that of the existing lens. This will nearly triple the amount of photons collected by the PMT during the detection interval, shifting the histograms to the right. The addition of a chiller system to the PMTs has also decreased the background counts from about 12 counts per second to 4 counts per second. This is likely limited by cosmic rays, but nonetheless it is a significant improvement. Overall, for our typical 800 microsecond detection time, the mean of bright distribution would move to about 27 photon counts. The discriminator threshold would then be changed to a value of 4 photon counts. The overlap between the bright state and dark state would increase, causing the discriminator method to be significantly worse than it was previously. However, by decreasing the detection time, the overlap between the histograms can be decreased. In fact, by cutting the detection time in half, roughly the same fidelity as current methods can be achieved. As mentioned above, having short, high fidelity quantum gates is necessary for efficient quantum information processing.

The incorporation of the adaptive likelihood method in addition to the introduction of a collection lens with a larger numerical aperture should prove to be an important resource for the realization of a quantum computer using trapped ions. Ions naturally have long coherence times, and by decreasing the detection gate while increasing the fidelity, fault tolerant quantum computation is one step closer.

APPENDICES

APPENDIX A

Mathematica Simulation Code

In Chapter V, we described a procedure used to generate random photons and estimate the theoretical error associated with the likelihood method. The relevant code for both the can be in the following pages with brief descriptions for each.

We begin by inspecting the bright state photon generation code. We first define the relevant atomic parameters. The arrival times table stores a time and the number of photons detected at that particular time. PhotonCount is a table that will keep track of Counter which is how many photons are generated in each experiment, and will be used to plot the histogram of the bright state. The flag will be used as a way to determine whether the ion has leaked to the dark state and the the other parameters have been previously defined.

Timebins = 100; ArrivalTimes = Table[0, {i, Timebins}]; \[Gamma]Bright = 10/.8; \[Gamma]Dark = 0.0000/.8; \[Gamma]Flip = .08/.8; \[Gamma]bg = 0.06/.8; Flag = 0; Counter = 0; NumExperiments = 10; PhotonCount = Table[0, {i, 25}];

[Tau]d = .8;

Now we present the code that will generate random photons. The first loop serves to run a large of number of experiments given by the parameter NumExperiments. The table called RandomArrivals is a list of random numbers that will be compared to the probability that a photon is generated in a particular time bin. If the random number exceeds the probability to generate a photon, then a photon is generated. The table RandomFlips is a list of random numbers that will be compared to the probability to flip in a given time bin. If the number is larger than that probability, the ion will flip and remain in the dark state for the rest of the detection time. At the end of each time bin, the ArrivalTimes table is modified to append the time bin and the number of photons generated in that time bin At the end of each experiment, PhotonCount is updated to reflect the number of photons generated in that particular experiment. Photon Count is then normalized and plotted in the form of a bar chart.

```
For[j = 1, j <= NumExperiments, j++,</pre>
  Flag = 0;
  Counter = 0;
  RandomArrivals = Table[Random[], {i, Timebins}];
  RandomFlips = Table[Random[], {i, Timebins}];
  For[i = 1, i <= Timebins, i++,</pre>
   Flag =
    If[Flag == 0,
     If[RandomFlips[[i]] > E^(-(\[Gamma]Flip) \[Tau]d/Timebins), 1,
      0], 1];
   ArrivalTimes[[i]] = {i*\[Tau]d/Timebins,
     If[RandomArrivals[[i]] >
       E^{(-(\[Gamma]Bright (1 - Flag) +
           Flag \[Gamma]Dark + \[Gamma]bg) \[Tau]d/Timebins), 1, 0]};
   Counter = If[ArrivalTimes[[i, 2]] == 1, Counter + 1, Counter];
   ];
  PhotonCount[[Counter + 1]] = PhotonCount[[Counter + 1]] + 1;
  ];
```

Following the same procedure as above, we now investigate the code used to generate photons by an ion initially in the dark state.

```
[Tau]d = 0.8;
\[Gamma]Bright = 10/.8;
\[Gamma]Dark = 0.000/.8;
\[Gamma]Flip = 1.3 0.05/.8 2/50 1.1;
\[Gamma]bg = 0.06/.8;
NumExperiments = 100;
Timebins = 100;
ArrivalTimes = Table[0, {i, Timebins}];
Counter = 0;
PhotonCount = Table[0, {i, 20}];
For[j = 1, j <= NumExperiments, j++,</pre>
  Counter = 0;
  RandomArrivals = Table[Random[], {i, Timebins}];
  RandomFlips = Table[Random[], {i, Timebins}];
  Flag = 0;
  For[i = 1, i <= Timebins, i++,</pre>
   Flag =
    If[Flag == 0,
     If[RandomFlips[[i]] > E^(-(\[Gamma]Flip) \[Tau]d/Timebins), 1,
      0], 1];
   ArrivalTimes[[i]] = {i*\[Tau]d/Timebins,
     If[RandomArrivals[[i]] >
       E^{(-(\[Gamma]Dark (1 - Flag) +
           Flag \[Gamma]Bright + \[Gamma]bg) \[Tau]d/Timebins), 1, 0]};
   Counter = If[ArrivalTimes[[i, 2]] == 1, Counter + 1, Counter];
  ];
  PhotonCount[[Counter + 1]] = PhotonCount[[Counter + 1]] + 1;
```

Notice that the flipping parameter has changed to reflect the flipping rate for the dark state.

Now, all relevant functions can be created in order to construct the likelihood function. The function z[j] gives the number of photons in the j^{th} time bin. Flip is the probability to flip in any given time bin and Flipbar represents the probability to not flip the entire detection time. Pdark/brightsim refer to the distributions described in chapter 5.

z[j_] := ArrivalTimes[[j, 2]];

Flip[\[Tau]d_, \[Gamma]flip_, Timebins_] :=

```
1 - E<sup>((-\[Gamma]flip*\[Tau]d)/Timebins);</sup>
```

Flipbar[\[Tau]d_, \[Gamma]flip_] := E^(-\[Gamma]flip*\[Tau]d);

```
Ldsim = Flipbar[\[Tau]d, \[Gamma]flipdark]*
```

Product[Pdarksim[z[n]], {n, 1, Timebins}] +

Flip[\[Tau]d, \[Gamma]flipdark, Timebins] Sum[

Product[Pdarksim[z[m]], {m, 1, j - 1}] Product[

Pbrightsim[z[k]], {k, j, Timebins}], {j, 1, Timebins}]

```
Lbsim = Flipbar[\[Tau]d, \[Gamma]flipbright] Product[
```

Pbrightsim[z[n]], {n, 1, Timebins}] +

```
Flip[\[Tau]d, \[Gamma]flipbright, Timebins] Sum[
```

Product[Pbrightsim[z[m]], {m, 1, j - 1}] Product[

Pdarksim[z[k]], {k, j, Timebins}], {j, 1, Timebins}]

Lrsim = Lbsim/Ldsim

After defining the likelihood function and ratio, we can now test the likelihood ratio and the discriminator method with a large number of experiments for different detection times. We begin by inspecting the detection error for an ion initialized in the bright state:

numExperiments = 1000;

 $\[Gamma]bg = 0.06/.8;$

Timebins = 100;

```
Timesteps = 50;
LikelihoodErrorTableBright = Table[0, {i, Timesteps}];
DiscriminatorErrorTableBright = Table[0, {i, Timesteps}];
discriminator = 1;
LikelihoodRatio = 0 ;
ArrivalTimes = Table[0, {i, Timebins}];
```

Everything is the same as before, except the LikelihoodErrorTableBright and DiscriminatorErrorTable-Bright refer to the number of incorrect inferences of the bright state for each detection time. Timesteps refers to the number of detection times that will be tested and the discriminator value is the comparison value for the discriminator method. Now the real simulation can be carried out with the following code:

```
For[j = 1, j <= Timesteps, j++,</pre>
```

```
[Tau]d = .1 + j 0.015;
LikelyHoodError\[Tau]d = 0;
DiscriminatorError\[Tau]d = 0;
For[m = 1, m <= numExperiments, m++,</pre>
Flag = 0;
Counter = 0;
RandomArrivals = Table[Random[], {i, Timebins}];
RandomFlips = Table[Random[], {i, Timebins}];
PhotonArrivalList = Table[0, {a, 1}];
For[i = 1, i <= Timebins, i++,</pre>
 Flag =
   If[Flag == 0,
    If[RandomFlips[[i]] >
      E^(-(\[Gamma]flipbright) \[Tau]d/Timebins), 1, 0], 1];
  ArrivalTimes[[i]] = {i*\[Tau]d/Timebins,
    If[RandomArrivals[[i]] >
      E^{(-(\[Gamma]Bright (1 - Flag) +
          Flag \[Gamma]Dark + \[Gamma]bg) \[Tau]d/Timebins), 1, 0]};
```

```
Counter = If[ArrivalTimes[[i, 2]] == 1, Counter + 1, Counter];
 ];
 Ldsim =
 Flipbar[\[Tau]d, \[Gamma]flipdark]*
    Product[Pdarksim[z[n]], {n, 1, Timebins}] +
  Flip[\[Tau]d, \[Gamma]flipdark, Timebins] Sum[
     Product[Pdarksim[z[m]], {m, 1, j - 1}] Product[
       Pbrightsim[z[k]], {k, j, Timebins}], {j, 1, Timebins}];
Lbsim =
 Flipbar[\[Tau]d, \[Gamma]flipbright] Product[
     Pbrightsim[z[n]], {n, 1, Timebins}] +
  Flip[\[Tau]d, \[Gamma]flipbright, Timebins] Sum[
     Product[Pbrightsim[z[m]], {m, 1, j - 1}] Product[
       Pdarksim[z[k]], {k, j, Timebins}], {j, 1, Timebins}];
Lrsim = Lbsim/Ldsim;
LikelihoodError\[Tau]d =
 If[Lrsim <= 1, LikelihoodError\[Tau]d + 1, LikelihoodError\[Tau]d];</pre>
 DiscriminatorError\[Tau]d =
 If[Counter <= discriminator, DiscriminatorError\[Tau]d + 1,</pre>
  DiscriminatorError\[Tau]d];
];
LikelihoodErrorTableBright[[j]] = {\[Tau]d,
 LikelihoodError\[Tau]d /numExperiments};
DiscriminatorErrorTableBright[[j]] = {\[Tau]d,
 DiscriminatorError\[Tau]d/numExperiments};
LikelihoodError\[Tau]d = 0;
DiscriminatorError\[Tau]d = 0;
```

```
];
```

Similarly, this can be carried out for the dark state.

```
Timesteps = 50;
\[Gamma]bg = 0.06/.8;
numExperiments = 1000;
LikelihoodErrorTableDark = Table[0, {i, Timesteps}];
DiscriminatorErrorTableDark = Table[0, {i, Timesteps}];
LikelihoodRatio = 0 ;
LikelyHoodError\[Tau]d = 0;
DiscriminatorError\[Tau]d = 0;
ArrivalTimes = Table[0, {i, Timebins}];
For[j = 1, j <= Timesteps, j++,</pre>
  [Tau]d = .1 + j 0.015;
  Timebins = 100;
  For[m = 1, m <= numExperiments, m++,</pre>
   Counter = 0;
   RandomArrivals = Table[Random[], {i, Timebins}];
   RandomFlips = Table[Random[], {i, Timebins}];
   Flag = 0;
   For[i = 1, i <= Timebins, i++,</pre>
    Flag =
     If[Flag == 0,
      If[RandomFlips[[i]] > E^(-(\[Gamma]flipdark) \[Tau]d/Timebins),
       1, 0], 1];
    ArrivalTimes[[i]] = {i*\[Tau]d/Timebins,
      If[RandomArrivals[[i]] >
        E^{(-(\backslash[Gamma]Dark (1 - Flag) +
            Flag \[Gamma]Bright + \[Gamma]bg) \[Tau]d/Timebins), 1,
       0]};
    Counter = If[ArrivalTimes[[i, 2]] == 1, Counter + 1, Counter];
```

```
];
```

```
Ldsim =
  Flipbar[\[Tau]d, \[Gamma]flipdark]*
    Product[Pdarksim[z[n]], {n, 1, Timebins}] +
   Flip[\[Tau]d, \[Gamma]flipdark, Timebins] Sum[
     Product[Pdarksim[z[m]], {m, 1, j - 1}] Product[
       Pbrightsim[z[k]], {k, j, Timebins}], {j, 1, Timebins}];
 Lbsim =
  Flipbar[\[Tau]d, \[Gamma]flipbright] Product[
     Pbrightsim[z[n]], {n, 1, Timebins}] +
   Flip[\[Tau]d, \[Gamma]flipbright, Timebins] Sum[
     Product[Pbrightsim[z[m]], {m, 1, j - 1}] Product[
       Pdarksim[z[k]], {k, j, Timebins}], {j, 1, Timebins}];
 Lrsim = Lbsim/Ldsim;
 LikelihoodError[Tau]d =
  If[Lrsim >= 1, LikelihoodError\[Tau]d + 1, LikelihoodError\[Tau]d];
 DiscriminatorError\[Tau]d =
  If[Counter > discriminator, DiscriminatorError\[Tau]d + 1,
   DiscriminatorError\[Tau]d];
];
LikelihoodErrorTableDark[[j]] = {\[Tau]d, LikelihoodError\[Tau]d /
  numExperiments};
DiscriminatorErrorTableDark[[j]] = {\[Tau]d,
  DiscriminatorError\[Tau]d/numExperiments};
LikelihoodError\[Tau]d = 0;
DiscriminatorError\[Tau]d = 0;
];
```

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