Trapped ions in the strong-excitation regime: Ion interferometry and nonclassical states

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The interaction of a trapped ion with a laser beam in the strong-excitation regime is analyzed. In this regime, a variety of nonclassical states of motion can be prepared either by using laser pulses of well defined area, or by an adiabatic passage scheme based on the variation of the laser frequency. We show how these states can be used to investigate fundamental properties of quantum mechanics. We also study possible applications of this system to build an ion interferometer. [S1050-2947(96)00108-4]

I. INTRODUCTION

In recent years, there has been growing interest in the preparation of nonclassical states of quantum systems in order to study fundamental properties of quantum mechanics. In the cavity QED context [1], for example, there are several proposals to prepare Fock states of the radiation field [2], or general superpositions of these states [3]. For single trapped ions, it has been shown how to prepare both Fock and squeezed states of the motion of an ion using a laser beam to excite an internal transition [4].

From the experimental point of view, the main difficulty in preparing nonclassical states in a given system is the presence of decoherence due to its coupling to external environments. In cavity QED this may be overcome by using high-Q cavities, where the coupling strength between the atoms and the cavity mode is of the order of or larger than both, the cavity loss rate and the spontaneous emission rate. With trapped ions, one can use an electric-dipole forbidden transition in order to avoid dissipation. In this last case, the quantum jump technique [5] allows one to peform measurements on the internal atomic state with a very high efficiency.

So far, all the proposals regarding the preparation of nonclassical states of motion of a single ion operate in the *low excitation regime*, whereby the Rabi frequency corresponding to the interaction of the ion with the laser Ω is much smaller than the trap frequency ν ($\Omega \ll \nu$). In this case, the Hamiltonian describing this interaction is very similar to the one that describes cavity QED, namely, the Jaynes–Cummings Hamiltonian. Thus, some of the quantum phenomena predicted in cavity QED, such as collapse and revival or vacuum Rabi splitting, have been predicted to exist for trapped ions under this regime [6,7]. Furthermore, the unique features of the ion-laser interaction has led to various proposals on how to prepare nonclassical states of motion, which have no analog in cavity QED. For example, Fock states can be prepared by observing quantum jumps in the internal state of the ion [7], or by sending short laser pulses with well defined area to the ion [8]. Coherent and squeezed states of motion can be prepared by bichromatic excitation of the ion [9].

In this paper we analyze the possibility of preparing nonclassical states of motion of a single ion in a completely different regime. We concentrate on the *strong excitation regime*, whereby the Rabi frequency describing the interaction of the ion with the laser is much larger than the trap frequency ($\Omega \gg \nu$). In this regime, the Hamiltonian describing such interaction is completely different from the one used in cavity QED, and therefore other phenomena can be investigated. In particular, we will show here how one can prepare quantum superpositions of two ''macroscopically'' distinct quantum states, such as

$$|\Psi\rangle = K(|\alpha\rangle_c + |-\alpha\rangle_c), \tag{1}$$

where K is a normalization constant, $|\alpha\rangle_c$ is a coherent state of the motion

$$|\alpha\rangle_c = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle, \tag{2}$$

and $|n\rangle$ denotes a Fock state with n phonons [10]. States (1) are usually called Schrödinger-cat states [11,12]. The fundamental properties of quantum-mechanical superpositions of states such as (1) have attracted due attention, and many schemes for their production have been proposed. In the cavity QED context, it has been shown [13] that during the collapse time, the state of the field is of the form (1). On the other hand, with a micromaser interaction one can also produce these states by sending atoms through a cavity [14]. The method we propose consists of two parts: first, using a laser one splits into two parts the ion wave function; then, one makes a measurement of the *internal* state of the ion in order to project its *external* state into (1). Thus the projection postulate of quantum mechanics is one of the ingredients of our method. We will show two different ways to prepare the state (1), one based on laser pulses and the other on the adiabatic change of the laser frequency during the interaction
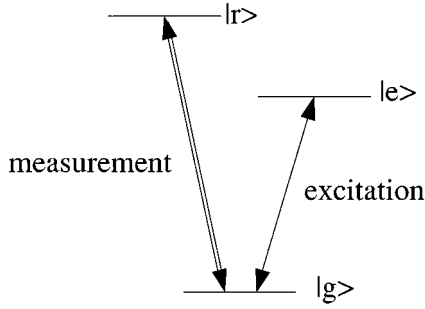
FIG. 1. Level scheme of the internal transitions of a trapped ion. Transition $|g\rangle \leftrightarrow |e\rangle$ is dipole forbidden, whereas transition $|g\rangle \leftrightarrow |r\rangle$ is allowed.

[15]. We will also show how to prepare linear superpositions of coherent states in more than one dimension, and how to distinguish between that state and an incoherent superposition of two coherent states. Finally, we will study how a Ramsey interferometer [16] can be built using these ideas.

This paper is organized as follows: first, in Sec. II we give a description of the strong-excitation regime, and find analytical approximations for the evolution of a single ion under this regime. In Sec. III we show how to prepare states of the form (1) in one and two dimensions, and in Sec. III we analyze how these states can be distinguished from statistical mixtures. The possibility of building an interferometer with trapped ions is analyzed in Sec. IV. A summary of the results of the paper is given in Sec. V.

II. STRONG-EXCITATION REGIME

In this section we analyze the interaction of a trapped ion with a laser beam in the strong-excitation regime. In this regime, the Rabi frequency for the laser-ion interaction is much larger than the trap frequency ($\Omega \gg \nu$), i.e., one can neglect the ion motion during the interaction. We will also assume that the laser is tuned to an electric-dipole forbidden transition $|g\rangle \leftrightarrow |e\rangle$ (see Fig. 1), and therefore we will neglect spontaneous emission. Under these conditions the problem becomes exactly solvable. Here we will study how the motion is modified after this interaction in two different situations: (i) the ion interacts with a laser pulse of well defined area; (ii) the frequency of the laser is varied adiabatically.

A. Excitation with laser pulses

Let us consider a single ion trapped in a three-dimensional harmonic potential. The ion interacts with a laser plane wave propagating along the x axis. In this configuration, only the motion along the x axis of the ion will be modified, so that we can treat this problem in one dimension. The Hamiltonian describing this situation is, in a rotating frame at the laser frequency ω_L ($\hbar = 1$),

$$H_\pm = \frac{\hat{p}_x^2}{2m} + \frac{1}{2}m\nu_x^2\hat{x}^2 - \frac{\delta}{2}\sigma_z + \frac{\Omega}{2}(\sigma_+ e^{\pm ik_L\hat{x}} + \sigma_- e^{\mp ik_L\hat{x}}).$$

$$(3)$$

Here, $\hat{x}$ and $\hat{p}_x$ are the position and momentum operators for the x coordinate of the ion, ν_x is the trap frequency along

this direction, and m is the ion mass. The sigmas are usual spin 1/2 operators describing the internal two–level transition $|g\rangle \leftrightarrow |e\rangle$, $\delta = \omega_L - \omega_0$ the laser detuning, $k_L = \omega_L/c$ the laser wave vector, and Ω the Rabi frequency. The subscripts ''+'' and (''−'') indicate that the laser plane wave propagates towards the positive (negative) values of x.

The evolution given by Hamiltonian (3) is easily described if we perform a unitary operation defined by

$$U_\pm = e^{\mp ik_L\hat{x}|e\rangle\langle e|}.$$

$$(4)$$

Using this operator, the states are defined as

$$|\tilde{\Psi}_\pm\rangle = U_\pm|\Psi\rangle,$$

$$(5)$$

whereas the Hamiltonian becomes

$$\tilde{H}_\pm \equiv U_\pm H_\pm U_\pm^\dagger = \nu_x a^\dagger a \pm i\nu_x\eta_x(a - a^\dagger)|e\rangle\langle e|$$

$$+ \nu_x\eta_x^2|e\rangle\langle e| - \frac{\delta}{2}\sigma_z + \frac{\Omega}{2}\sigma_x,$$

$$(6)$$

where $\sigma_x = \sigma_+ + \sigma_-$, and we have expressed the position and momentum operators in terms of the annihilation and creation operators of the harmonic oscillator, a and $a^\dagger$, respectively, and in terms of the Lamb-Dicke parameter $\eta_x = k_L/(2m\nu_x)^{1/2}$. The new terms appearing in Hamiltonian (6) correspond to the Doppler recoil energy of the excited internal state (i.e., when one photon is absorbed).

Hamiltonian (6) can be simplified in the strong-excitation regime $\Omega \gg \nu_x$. Let us consider a laser pulse whose duration τ fulfills the following inequality

$$\nu_x\tau\max(\bar{n}, \eta_x^2) \ll 1,$$

$$(7)$$

where $\bar{n} = \langle a^\dagger a \rangle$. Note that in the strong-excitation limit, this interaction time can correspond to pulses with an area $\Omega\tau \sim \pi$. Under condition (7), the first terms in Hamiltonian (6) will practically not affect the evolution. In this case, and assuming for simplicity that the detuning is zero ($\delta = 0$), the Hamiltonian reduces to $\tilde{H} = 1/2\Omega\sigma_x$ (for both cases, $\tilde{H}_\pm$) [17]. The evolution of any initial state can be easily derived, obtaining

$$|\Psi_\pm(\tau)\rangle = U_\pm^\dagger e^{-i(1/2)\Omega\sigma_x\tau} U_\pm|\Psi(0)\rangle.$$

$$(8)$$

Note that the evolution given by H_- can be obtained from that given by H_+ by simply exchanging $\eta_x \to -\eta_x$.

In the following we will need to know the evolution for the case where the initial state of motion along the x direction is a coherent state $|\alpha\rangle_c$. In this situation, Eq. (8) gives

$$|g\rangle|\alpha\rangle_c \to A|g\rangle|\alpha\rangle_c + B|e\rangle|\alpha \pm i\eta_x\rangle_c,$$

$$(9a)$$

$$|e\rangle|\alpha\rangle_c \to A^*|e\rangle|\alpha\rangle_c - B^*|g\rangle|\alpha \mp i\eta_x\rangle_c,$$

$$(9b)$$

where the upper (lower) sign corresponds to a laser pulse propagating towards the positive (negative) values of the x axis. For reasons that will become clear in the next subsection, we have defined in (9)

$$A = \cos(\Omega\tau/2),$$

$$(10a)$$

$$B = -i\sin(\Omega\tau/2). \tag{10b}$$

Finally, in this section we have assumed square laser pulses. For any other kind of pulse, formulas (9) are still valid, but now one has to replace $\Omega\tau$ by $\int_0^\tau \Omega(t)dt$.

B. Excitation by adiabatic passage

In this subsection we will analyze a different method of obtaining results that are similar to those of the preceding subsection. Here, instead of using a laser pulse to modify the internal and external state of the ion, we will consider a process in which the laser frequency is changed adiabatically.

Let us consider then the same situation as before. A trapped ion interacts with a laser plane wave propagating along the x direction. The Hamiltonian describing this situation, after applying the unitary operation (4) is given by Eq. (6), although now it is time dependent (through the time dependence of the detuning). As before, for short interaction times τ fulfilling Eq. (7), the Hamiltonian can be simplified to

$$\widetilde{H}(t) = -\frac{\delta(t)}{2}\sigma_z + \frac{\Omega}{2}\sigma_x. \tag{11}$$

At a given time t, the instantaneous eigenstates of this Hamiltonian are the well known dressed states

$$|+(t)\rangle = \cos[\theta(t)]|e\rangle + \sin[\theta(t)]|g\rangle, \tag{12a}$$

$$|-(t)\rangle = -\sin[\theta(t)]|e\rangle + \cos[\theta(t)]|g\rangle, \tag{12b}$$

with corresponding eigenvalues

$$E_\pm(t) = \pm\tfrac{1}{2}\sqrt{\delta(t)^2 + \Omega^2}, \tag{13}$$

and where $\cot[2\theta(t)] = -\delta(t)/\Omega$ $(0 \leq 2\theta < \pi)$.

By changing the detuning adiabatically from $\delta_0 = \delta(0)$ to $\delta_\tau = \delta(\tau)$ the internal state of the ion will follow (approximately) the evolution of these dressed states, i.e.,

$$|\pm(0)\rangle \rightarrow e^{\mp i\epsilon}|\pm(\tau)\rangle, \tag{14}$$

where

$$\epsilon = \int_0^\tau E_+(t)dt. \tag{15}$$

The condition for the adiabatic passage to be valid (14) can be easily estimated [18]. For example, assuming that the rate of change of δ is constant, one finds

$$|\delta_0 - \delta_\tau| \ll \Omega^2\tau. \tag{16}$$

Using Eqs. (4) and (14), we find (9) for the evolution of an ion initially in a coherent state of motion, where now

$$A = \cos(\epsilon)\cos(\theta_0 - \theta_\tau) + i\sin(\epsilon)\cos(\theta_0 + \theta_\tau), \tag{17a}$$

$$B = \cos(\epsilon)\sin(\theta_0 - \theta_\tau) - i\sin(\epsilon)\sin(\theta_0 + \theta_\tau), \tag{17b}$$

and $\theta_0 = \theta(0)$ $[\theta_\tau = \theta(\tau)]$. Thus, by choosing appropriately the initial and final detunings and provided the adiabatic con-

dition (16) is fulfilled one can achieve by adiabatic passage the same effect as using laser pulses.

III. SUPERPOSITIONS OF COHERENT STATES

In this section we show how superpositions of coherent states (Schrödinger cats) of the ion motion can be prepared using a laser beam in the strong-excitation regime. We will analyze both the one-dimensional (1D) and the two-dimensional (2D) cases.

A. One dimension

Let us assume that after sideband cooling [19], we have the ion in the ground state of both, the internal and external degrees of freedom $(|g\rangle|0\rangle)$. In this subsection we discuss two ways of preparing states of the form (1) in one-dimension (x), one based on laser pulses and the other on adiabatic passage by varying the laser frequency. For the sake of a simple notation, we will drop the subscript x in all the states and formulas when we deal with one-dimensional problems.

1. Pulses

A simple way to prepare the state (1) using laser pulses consists of the following three steps:

(i) Excite the ion with a $\pi/2$ pulse $(\Omega\tau = \pi/2)$ using a laser propagating towards the negative values of the x axis. According to (9), this pulse will perform the following transformation:

$$|g\rangle|0\rangle_c \rightarrow \frac{1}{\sqrt{2}}(|g\rangle|0\rangle_c - i|e\rangle|-i\eta\rangle_c), \tag{18}$$

where $|\alpha\rangle_c$ denotes the coherent state (2).

(ii) Excite the ion with another $\pi/2$ pulse $(\Omega\tau = \pi/2)$, but now using a laser beam propagating in the opposite direction. The state of the ion will become

$$\tfrac{1}{2}[(|0\rangle_c - |-2i\eta\rangle_c)|g\rangle - i(|i\eta\rangle_c + |-i\eta\rangle_c)|e\rangle]. \tag{19}$$

(iii) Measure the internal state of the ion using the quantum jump technique [5]. To do this one can drive the ion with a different laser beam on resonance with an electric-dipole allowed transition $|g\rangle \leftrightarrow |r\rangle$ (see Fig. 1). In case fluorescence is observed, the state of the ion will be projected onto the part of the wave-function (19) that contains the state $|g\rangle$. On the contrary, if no fluorescence is observed, it will be projected onto the state

$$|\Psi_{sc}\rangle = K(|i\eta\rangle_c + |-i\eta\rangle_c)|e\rangle, \tag{20}$$

where K is a normalization constant.

State (20) is already of the form (1), i.e., similar to that studied in cavity QED [13,14] (whereby the motion of the ion corresponds to the cavity mode). Note that if fluorescence is observed, the external state of the motion will be completely modified due to the photon recoil acquired by the ion in each absorption spontaneous emission cycle, and therefore no such superposition of coherent states will be produced. Therefore, an experiment based on these steps will be successful half of the times it is carried out (those in which no fluorescence is observed).
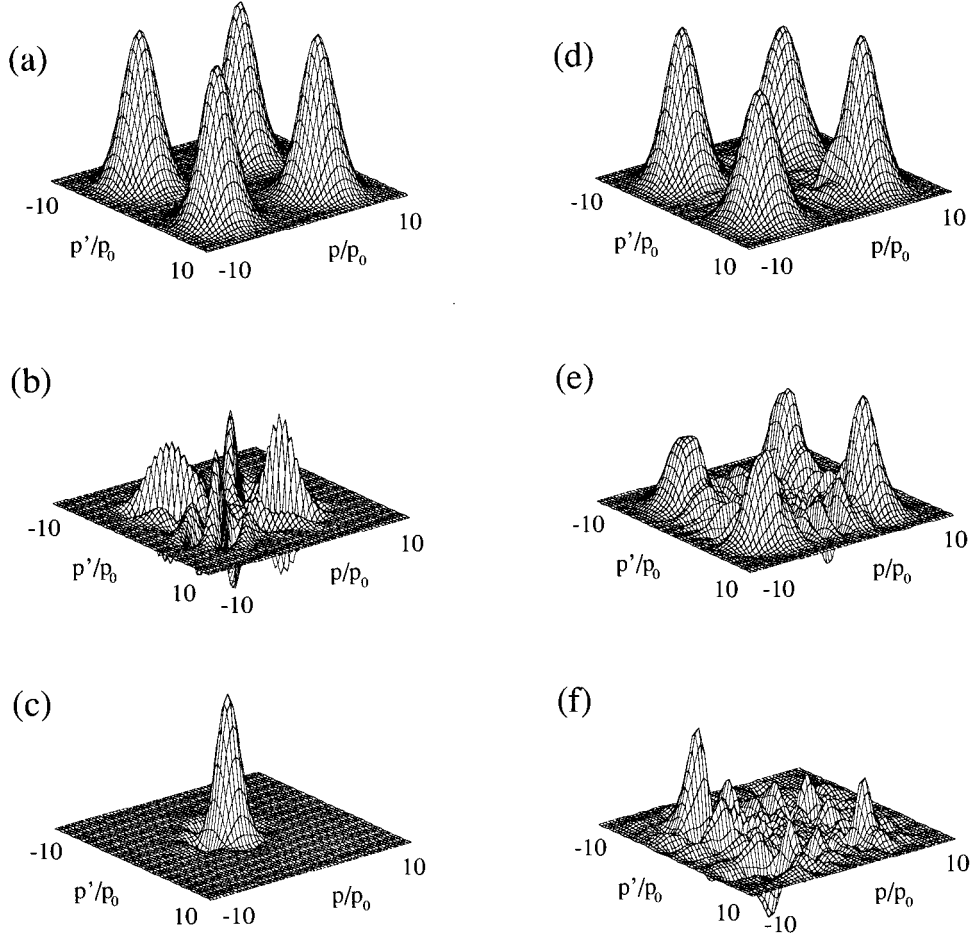
FIG. 2. Real part of the density operator in momentum representation $\langle p|\rho|p'\rangle$ in arbitrary units [note that the axes are rescaled in terms of $p_0=(m\nu/2)^{1/2}$], after the preparation of the superposition state using laser pulses. (a)–(c) correspond to $\eta=0.5$ and $n=2$, whereas (d)–(f) correspond to $\eta=2.5$ and $n=0$. Here Ω/ν: 100 (a,d), 10 (b,e), and 1 (c,f).

On the other hand, in order to consider state (20) as a Schrödinger cat it should be macroscopic (in the sense that one can observe both states $|\alpha\rangle$ and $|-\alpha\rangle$ individually [10]). However, there are two reasons why the state (20) cannot be directly observable as it stands: (a) its probability distribution in position representation has only one peak centered at $\langle\hat{x}\rangle=0$, and therefore the two parts of such a state ($|i\eta\rangle$ and $|-i\eta\rangle$) cannot be distinguished by direct observation; (b) since the ion is in its internal excited state, it cannot be observed by shining light on the $|g\rangle\leftrightarrow|r\rangle$ transition. This second problem (b) can be easily solved if, just after the state (1) is produced, one applies a laser π pulse along the z direction, that transforms the excited state into the ground state. Note in order not to modify the state of the ion with this last pulse it is necessary for the ion to be confined in the Lamb–Dicke limit in the z direction, i.e., $\eta_z=k_L/(2m\nu_z)^{1/2}\ll1$. This may be the case, for example, in a linear ion trap, whereby the transverse directions have trap frequencies much larger than along the axial direction [20]. The first problem (a) can be solved by noting that the free evolution of a coherent state is given by $|\alpha(t)\rangle=|e^{-i\nu t}\alpha(0)\rangle$ and, therefore, if after producing the state (1) one waits for a time $t=\pi/(2\nu)$, the state will become

$$|\Psi_{sc}\rangle=K(|\eta\rangle_c+|-\eta\rangle_c)|e\rangle. \tag{21}$$

This state has two maxima (in position representation) centered at $\langle\hat{x}\rangle=\pm2\eta/\sqrt{2m\nu}$, respectively (each of these maxima correspond to the states $|\pm\eta\rangle$, respectively). Furthermore, in order to be able to observe these two states the corresponding peaks must be spatially separated by more than a wavelength, which requires $\eta^2>\pi/2$. For tight traps, that is, for values of η not fulfilling this condition, one can proceed as follows. After step (i), one applies a sequence of π pulses from the left and from the right (n pulses in each direction), in an alternating way [21]. It is easy to check that the state after step (iii) and free evolution will be

$$|\Psi_{sc}\rangle=K[|(2n+1)\eta\rangle_c+|-(2n+1)\eta\rangle_c]|e\rangle. \tag{22}$$

For these states, the number of pulses required to observe distinguishable (macroscopic) states is $(2n+1)^2>\pi/(2\eta^2)$. Note also that now in condition (7) it is the total time corresponding to all the pulses the one that enters.

In order to illustrate the effectiveness of the method presented here, we have plotted in Fig. 2 the state after step (iii) for several values of Ω/ν and η. To produce these plots, we have solved numerically the evolution equations of the ion
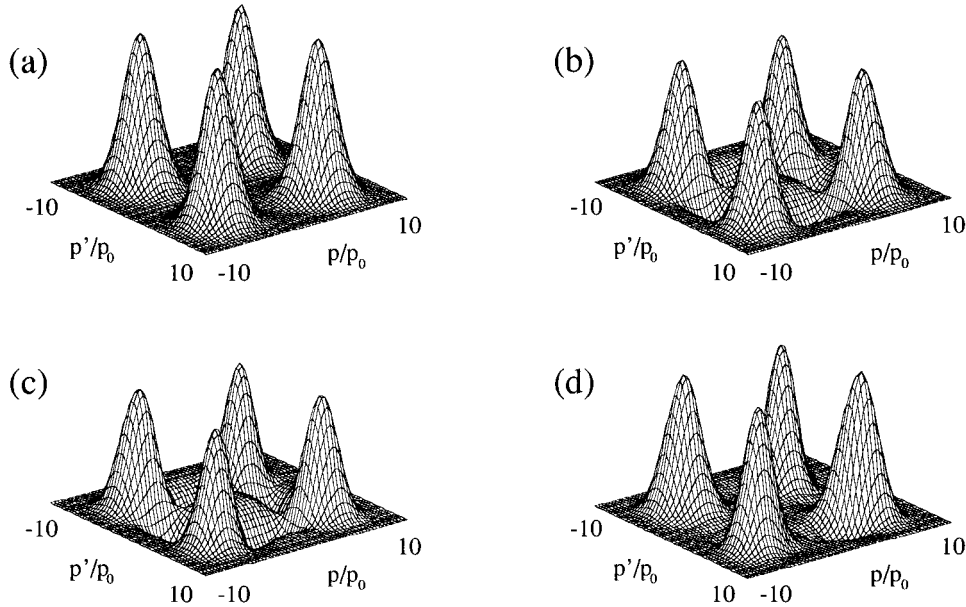
FIG. 3. Real part of the density operator in momentum representation $\langle p|\rho|p'\rangle$ in arbitrary units [note that the axes are rescaled in terms of $p_0=(m\nu/2)^{1/2}$], after the preparation of the superposition state using adiabatic passage. Here, $\Delta/\Omega=10$, $\Omega/\nu=100$, $\eta=0.5$, $n=2$, and (a) $\Omega\tau=40$; (b) $\Omega\tau=50$; (c) $\Omega\tau=60$; (d) $\Omega\tau=70$.

using the exact Hamiltonian (3). The figures display the real part of the density operator in momentum representation $\langle p|\rho|p'\rangle$ (note that the axes are rescaled in terms of $p_0=\sqrt{m\nu/2}$). Figures 2(a), 2(b), and 2(c) correspond to $\eta=0.5$ and $n=2$ (i.e., two intermediate π pulses in each direction), whereas Figs. 2(d), 2(e), and 2(f) correspond to $n=0$ with $\eta=2.5$. For $\Omega/\nu=100$ [Figs. 2(a) and 2(d)] the method works almost ideally, since condition (7) is satisfied. There are four peaks in the plots, two of them corresponding to the diagonal parts of the density operator ($|i\eta\rangle\langle i\eta|$ and $|-i\eta\rangle\langle -i\eta|$), and the other corresponding to the coherences ($|-i\eta\rangle\langle i\eta|$ and $|i\eta\rangle\langle -i\eta|$). These last two peaks are the ones ensuring that the state is a truly pure state, as opposed to a statistical mixture in which these two peaks will not show up. As soon as the ratio Ω/ν is decreased, so that condition (7) is not satisfied, this four peak structure disappears. Note that the plots corresponding to $n=2$ are much more sensitive to this condition than those with $n=0$. On the other hand, we have checked that they are more robust with respect to mismatches in the pulse areas as well.

2. Adiabatic passage

Instead of using controlled laser pulses, one can use the adiabatic passage technique to produce the superposition of coherent states. This technique has the advantage that it is not sensitive to the specific values of the parameters characterizing the laser-ion interaction (such as interaction time, Rabi frequencies, etc.). The process is very similar to the one explained above for pulses. It consists of three steps:

(i) A laser beam propagating towards the negative values of the x axis is directed to the ion. The detuning is switched adiabatically from $\delta_0=-\Delta$ to $\delta_\tau=0$, with $\Delta\gg\Omega$. In this case, according to (17) the state of the ion will become ($\theta_0=0,\theta_\tau=\pi/4$)

$$|g\rangle|0\rangle_c\rightarrow\frac{1}{\sqrt{2}}(|g\rangle|0\rangle_c-|e\rangle|-i\eta\rangle_c).\qquad(23)$$

(ii) A laser beam propagating towards the positive values of x is directed to the ion. The detuning is switched adiabatically from $\delta_0=0$ to $\delta_\tau=\Delta$, again with $\Delta\gg\Omega$. The state of the ion after this step will be ($\theta_0=\pi/4,\theta_1=\pi/2$)

$$\tfrac{1}{2}[e^{-i\epsilon}(|0\rangle_c-|-2i\eta\rangle_c)|g\rangle-e^{i\epsilon}(|i\eta\rangle_c+|-i\eta\rangle_c)|e\rangle].\qquad(24)$$

(iii) Measure the internal state of the ion using the quantum jump technique [5]. If no fluorescence is observed, the external state of the ion will be projected onto

$$|\Psi_{sc}\rangle=K(|i\eta\rangle_c+|-i\eta\rangle_c)|e\rangle.\qquad(25)$$

As before, one can include intermediate steps between (i) and (ii) to achieve a larger extension between the peaks of the probability distribution. To this aim, one can perform sequences of excitations, using counterpropagating lasers: first, changing the detuning from $\delta_0=0$ to $\delta_1=\Delta$, and then from $\delta_0=-\Delta$ to $\delta_1=0$. By performing this operation n times (in each direction), the final state of the ion (after measurement) will be

$$|\Psi_{sc}\rangle=K(|(2n+1)i\eta\rangle_c+|-(2n+1)i\eta\rangle_c)|e\rangle.\qquad(26)$$

In Fig. 3 we have plotted the real part of $\langle p'|\rho|p\rangle$ for the state arising after step (iii), by solving numerically the evolution equations using the full Hamiltonian (3). Here, $\eta=0.5$, and the number of intermediate steps is $n=2$. We have taken $\Delta=10\Omega$ and $\Omega/\nu=100$. The times for each adiabatic passage step are $\tau=40$, 50, 60, and 70 Ω^{-1} [Figs. 3(a), 3(b), 3(c), and 3(d), respectively]. Note that with these parameters the conditions of no motion during the interaction (7) and of adiabatic passage (16) are nearly fulfilled. By

comparing these four plots, one sees that in all of them four peaks show up, corresponding to the diagonal part and the coherences. However, in some of the plots there is an extra wide peak in the center, which does not disappear even if the interaction time is taken to be longer (in order to improve the adiabatic passage requirements). In fact, we have checked numerically that the peak appears nearly periodically as the time τ is increased. The reason for this peak is related to the fact that Δ is finite in the plots. Using relations (17) it can be easily shown that the four peaks (in $\langle p'|\rho|p\rangle$) oscillate due to the dynamical phase ϵ, and the amplitude of the oscillations is of the order of $n\Omega/\Delta$, where n is the number of intermediate steps. The best results are obtained for $n=0$, where there is no such oscillation; but this requires a large value for η in order for the two locations of the ion to be observable. This may be realized with a linear ion trap, for which the motion along the axial direction can have $\eta>1$. Summarizing, in order to prepare a state of the form (1) using adiabatic passage with intermediate laser pulses, apart from conditions (7) and (16), it is required that $\Delta\gg\Omega$. Note that this last condition is somehow difficult to achieve experimentally, since as one increases Δ, a longer time is required to fulfill the adiabatic condition, which may require a very small trap frequency in order not to violate condition (7).

B. Two dimensions

In this subsection we generalize the above methods to show how superpositions of coherent states in two or more dimensions can be generated. The basic idea is to prepare first a superposition state along a given direction following the steps presented in the previous subsections, and afterwards to send pulses along a perpendicular direction in order to push the wave packets and obtain a circular motion. The steps required to implement a superposition of coherent states in the x-y dimensions are the following:

(i) After sideband cooling [19], prepare the state (1) in the x direction as indicated in the preceding subsection. Then wait for a time $\tau=\pi/(2\nu)$. The state of the ion will be

$$|\Psi_{sc}\rangle=K(|\eta\rangle_x+|-\eta\rangle_x)|0\rangle_y|e\rangle. \qquad (27)$$

(ii) Drive the ion with a π pulse using a laser propagating along the y direction. The state will become

$$|\Psi_{sc}\rangle=K(|\eta\rangle_x+|-\eta\rangle_x)|-i\eta\rangle_y|g\rangle. \qquad (28)$$

State (28) is composed of two coherent wave packets each of them propagating along a circle in the x-y plane (provided the trap frequencies $\nu_x=\nu_y$), but in opposite directions. Note that the internal state of the ion after the pulse sequence is $|g\rangle$, and therefore it is ready to be detected by quantum jumps. Obviously, for a trap with a small value of η one can add intermediate π pulses in both directions x and y in order to make a larger circle radius. On the other hand, one can use the adiabatic passage technique instead of using pulses, utilizing the same procedure.

In Fig. 4 we have plotted snapshots of the probability distribution for the ion in position representation $P(x,y)=\langle x|\langle y|\rho|y\rangle|x\rangle$. We have solved numerically the evolution using the exact Hamiltonian, including both dimensions x and y. We have taken $\eta=0.5$, and $n=2$ in each

dimension, and $\Omega=300\nu$ ($\nu_x=\nu_y\equiv\nu$). The different figures correspond to the state of the ion: (a) initial state; (b) after step (ii); (c) after a time $\tau=\pi/(4\nu)$; (d) after a time $\tau=\pi/(2\nu)$; (e) state after a time $\tau=3\pi/(4\nu)$; (f) after a time $\tau=40\pi/\nu$ (i.e., after 20 round trips). As expected, the ion performs round trips without practically modifying the structure of the wave packets. Note that the figures correspond to the diagonal part of the density operator, and therefore one could have the ion in a statistical mixture with exactly the same distribution. However, the fact that the state is a pure state makes it possible to observe quantum interferences, as will be shown in the next sections. Finally, in these figures it can be observed that there is a small dephasing of the wave packets [compare Figs. 4(c) and 4(e)]. This is due to the fact that the pulses have a finite duration, and during this time the wave packets evolve slightly [that is, conditions (7) are not exactly fulfilled]. However, most parts of this dephasing can be controlled, since they correspond to the free evolution with the harmonic oscillator potential for a time equal to the pulse duration.

IV. PURE STATES VERSUS STATISTICAL MIXTURES

One of the crucial predictions of quantum mechanics is the possibility of having coherent superpositions of certain states, as is the case for a Schrödinger-cat state. Many of the intriguing features of quantum mechanics rely on this property. Thus, it would be highly desirable to have a way to check experimentally whether the final state is a pure state of the form (20) or an incoherent superposition of the form

$$\rho\propto(|\eta\rangle_c\langle\eta|+|-\eta\rangle_c\langle-\eta|)|e\rangle\langle e|. \qquad (29)$$

Similarly as for the pure state (20), in this statistical mixture the ion is either to the right (state $|\eta\rangle$) or to the left (state $-|\eta\rangle$) in the x axis.

Let us state this problem in a different (but equivalent) way. Suppose we have our ion in the state (29). Consider the following experiment consisting of four steps:

(i) We measure if the ion is to the right. This can be done by using the quantum jump technique [5] with a laser focused on the right side only (after changing the state of the ion in the right-hand side from $|e\rangle$ to $|g\rangle$, as indicated in Sec. III, i.e., using an auxiliar laser propagating along the z direction). If we do not detect the ion, then we know that the ion is in the left side, and its state is

$$|\Psi\rangle=|-\eta\rangle_c|e\rangle. \qquad (30)$$

(ii) Now we wait for a time $\tau=\pi/(2\nu)$, so that the state of the ion will be $|i\eta\rangle|e\rangle$, and its position will be centered around $x=0$.

(iii) We send two $\pi/2$ pulses along the x direction, the first propagating towards the negative values of x while the second propagating in the opposite direction. The state of the ion after this pulse sequence can be easily calculated using (9), resulting in

$$|\Psi\rangle=K[|e\rangle(|i\eta\rangle_c-|3i\eta\rangle_c)-i|g\rangle(|0\rangle_c+|2i\eta\rangle_c)]. \qquad (31)$$

(iv) We measure the state of the ion using the quantum jumps technique. Obviously, we will obtain that the probability of measuring the ion in its ground state is 1/2.
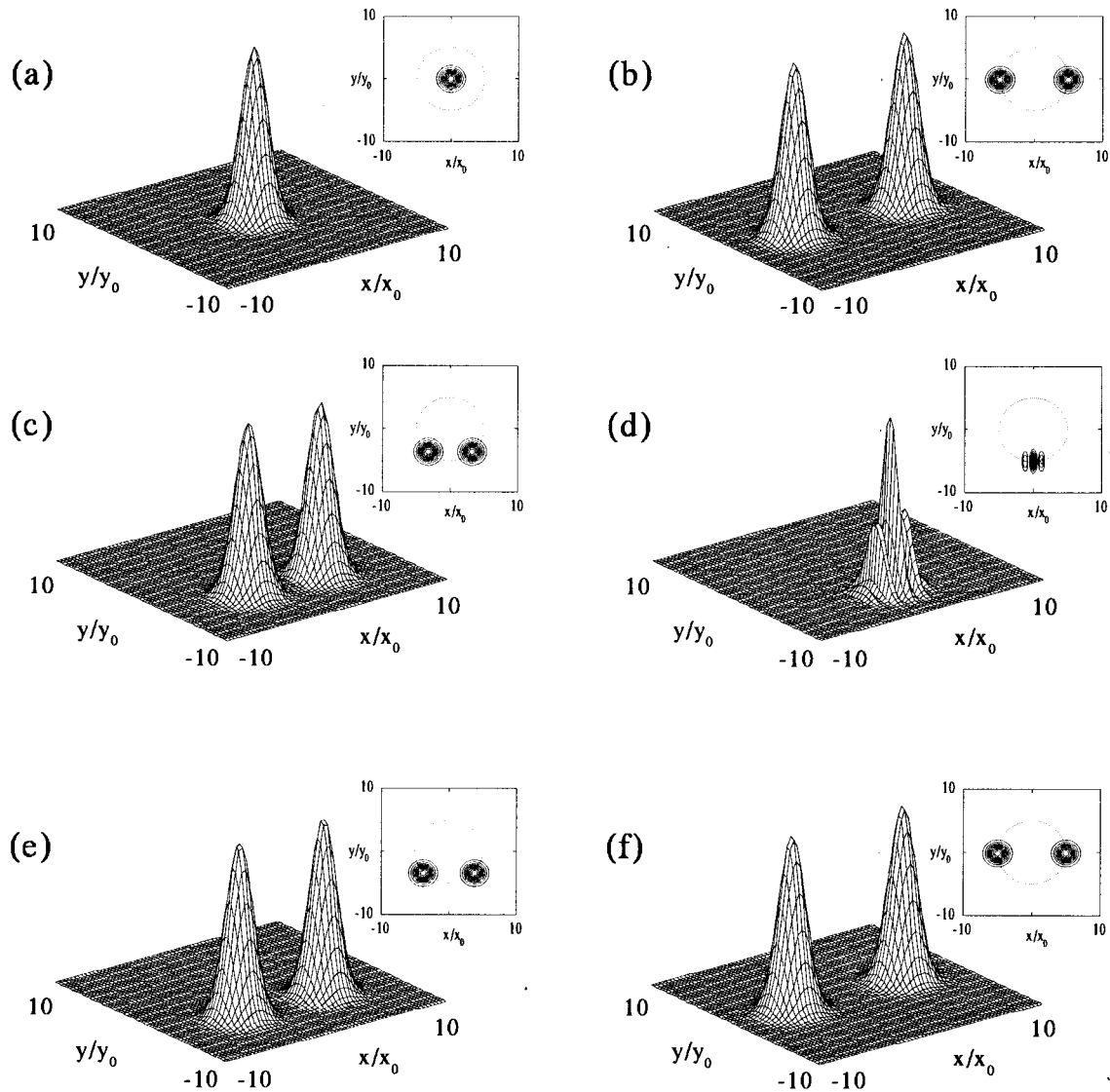
FIG. 4. Snapshots of the probability distribution in position representation in two dimensions $\langle x|\langle y|\rho|y\rangle|x\rangle$ in arbitrary units [note that the axes are rescaled in terms of $x_0 = y_0 = 1/(2mnu)^{1/2}$]. Here $\Omega/\nu = 300$, $\eta = 0.5$, $n = 2$, and (a) initial state; (b) just after the superposition state preparation; (c) after $\nu\tau = \pi/4$; (d) after $\nu\tau = \pi/2$; (e) after $\nu\tau = 3\pi/4$; (f) after $\nu\tau = 40\pi$.

We could perform the same experiment again, but now in step (i) we measure if the ion is to the left. If we do not detect the ion, we will conclude that the state of the ion is

$$|\Psi\rangle = |\eta\rangle_c|e\rangle. \tag{32}$$

Following the same steps as before (ii) and (iii), the state of the ion will be

$$|\Psi\rangle = K[e\rangle(|-i\eta\rangle_c - |i\eta\rangle_c) - i|g\rangle(|0\rangle_c + |-2i\eta\rangle_c)]. \tag{33}$$

Again, if we measure the state of the ion we will measure the ground state with a probability 1/2.

Summarizing, we could state that regardless of the position of the ion (i.e., both if it is to the left or to the right), after performing steps (ii) and (iii), we will detect half of the times the ion in the ground state, and the other half in the

excited state. However, if we take the state (20) and calculate what happens after applying steps (ii) and (iii), we will obtain the state

$$|\Psi\rangle = K|e\rangle(|-i\eta\rangle_c - |3i\eta\rangle_c) - i|g\rangle(2|0\rangle_c + |-2i\eta\rangle_c$$
$$+ |2i\eta\rangle_c). \tag{34}$$

With this state, the probability of detecting the atom in the ground state is 3/4.

Obviously, there is a catch in the above argument. In order to predict that whatever we have for the ion we will find it in the ground state with probability 1/2, we have used a property which is foreign to quantum mechanics, namely, realism [22]. We all know that quantum mechanics is not a realist theory, and therefore the apparent contradiction that we have explained in the previous paragraph shows this fact. State (20) is a state that can only be described by quantum mechanics, and there is no classical (realist) analog. Note

that the state (29) can be indeed described by a realist theory, and therefore if we perform steps (ii) and (iii) on it we will detect the ion in its ground internal state with probability 1/2. Therefore, the experiment outlined in this section [steps (ii) and (iii)] allows one to distinguish between a pure state and a statistical mixture, since the results for the probability of detecting the ground state of the ion are different.

V. ION INTERFEROMETER

In this section we analyze the possibilities of building a Ramsey interferometer [16] based on the ideas we have used to prepare superpositions of coherent states. We will analyze the case in which laser pulses are used instead of adiabatic passage, although both techniques can be applied with similar results.

Consider a single ion in a trap after being laser cooled (via sideband cooling) to its ground state $|g\rangle|0\rangle$. An atom interferometer (in one dimension) may be constructed as follows.

(i) Apply a $\pi/2$ pulse in the x direction such that the state of the ion becomes

$$|\Psi\rangle = \tfrac{1}{2}(|g\rangle|0\rangle_c - i|e\rangle|i\eta\rangle_c). \tag{35}$$

Then, wait for a time $\tau = \pi/(2\nu)$ so that the ion wave function (in position representation) splits into two different wave packets

$$|\Psi\rangle = \tfrac{1}{2}(|g\rangle|0\rangle_c - i|e\rangle|\eta\rangle_c). \tag{36}$$

Note that as before, for small values of η one can use intermediate pulses to split the wave packet further.

(ii) Apply a field to the wave packet that is centered to the right. For example, assume that we transform the state $|e\rangle \rightarrow \cos(\alpha)|e\rangle - i\sin(\alpha)|g\rangle$. In this particular case, the wave function will become

$$|\Psi\rangle = \tfrac{1}{2}[|g\rangle|0\rangle_c - i(\cos(\alpha)|e\rangle - i\sin(\alpha)|g\rangle)|\eta\rangle_c]. \tag{37}$$

Then, wait again for a time $\tau = \pi/(2\nu)$ so that the ion wave function (in position representation) is centered around $\langle x\rangle = 0$

$$|\Psi\rangle = \tfrac{1}{2}[|g\rangle|0\rangle_c - i(\cos(\alpha)|e\rangle - i\sin(\alpha)|g\rangle)|-i\eta\rangle_c]. \tag{38}$$

(iii) Apply a $\pi/2$ pulse to the ion in the opposite direction as in step (i). The state of the ion then will become

$$|\Psi\rangle = \tfrac{1}{2}(|g\rangle\{[1-\cos(\alpha)]|0\rangle_c - \sin(\alpha)|-i\eta\rangle_c\}$$
$$-i|e\rangle\{[1+\cos(\alpha)]|-i\eta\rangle_c - \sin(\alpha)|-2i\eta\rangle_c\}). \tag{39}$$

(iv) Measure the state of the ion using the quantum jumps technique. One finds that the probability of measuring the excited internal state is $P_e = \cos^2(\alpha/2) = 1 - P_g$. So, depending on the phase α this probability varies as in Ramsey spectroscopy.

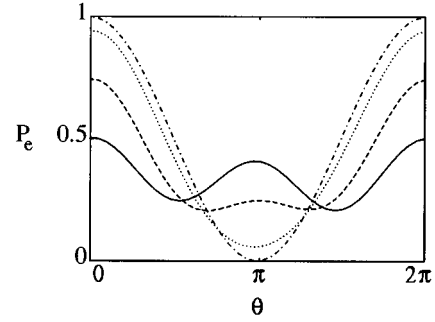In Fig. 5 we have plotted the probability of finding the ion in the excited state as a function of the phase α, calculated

FIG. 5. Excited-state population P_e as a function of the phase α (see text for explanation). Here, $\eta=2.5$, Ω/ν: 1 (solid line); 4 (dashed line); 10 (dotted line); 100 (dash-dotted line).

numerically using the exact Hamiltonian. Here $\eta=2.5$ and the different curves correspond to $\Omega/\nu=1$, 4, 10, and 100 [solid, dashed, dotted, and dash-dotted lines, respectively]. In the strong-excitation regime (dash-dotted line), the visibility of the fringes is nearly one. However, as the ratio Ω/ν decreases, the visibility is getting smaller and smaller, and even the curve is distorted. The reason is that the condition (7) is not satisfied anymore.

This kind of interferometer is very similar to the well-known Ramsey spectroscopy. However the main difference is that the phase shift here can be nonlocal, in the sense that one can do something different to the different wave packets, as is the case in atom interferometry [23]. Thus, a technique like this could be useful to measure gradients of fields, since the phase shift on each wave packet would depend on its corresponding position. Apart from that, following the lines of Sec. III B one can generalize this to two dimensions. In this case, one would have two wave packets going around in a circle but in opposite directions. An interferometer like this might be used to measure the Sagnac effect, where the phase acquired by one of the wave packets is different to that of the other when the whole ion trap rotates at a given frequency.

VI. CONCLUSIONS

In this paper we have analyzed the interaction of a single trapped ion with a laser beam in the strong-excitation limit. We have considered two situations: first, the ion interacts with pulses of a well defined area; second, the frequency of the laser changes adiabatically. We have shown under which limits these two situations give the same result regarding the modification of the ion motion during the excitation. Furthermore, Schrödinger cats [10] of the ion motion can be prepared in an ion trap using both methods. We have illustrated these methods with numerical calculations which display the preparation of superpositions of coherent states both in one and two dimensions. We have shown how one can check the purity of these states, by relating this problem to the one regarding the different predictions of (macro)realist's theories and quantum mechanics. Finally, a trapped ion can be used as an inteferometer by splitting the wave function and looking at the interferences in the internal states.

The technique presented here can be easily generalized to include the preparation of more general nonclassical states, including linear superposition of multiple coherent states

with the same amplitude and different phases. On the other hand, we plan to study in detail the problem of how the Schrödinger-cat state dissipates in time in the presence of decoherence due to spontaneous emission [24].

In order to observe experimentally the behavior predicted in this paper it is necessary to fulfill the conditions given in Sec. II, i.e., basically to work in the strong-excitation regime. This regime may be difficult to achieve, since one is using an electric-dipole forbidden transition and therefore the laser intensity required to reach such a regime must be very high. Alternatively, one could work with small trap frequencies (i.e., out of the Lamb-Dicke limit), which would make it easier to perform local observations. However, laser cooling to the ground state of the trapping potential has not been observed yet in this regime. One possible way to avoid this drawback is to open the trap adiabatically (in order to decrease the trap frequency, and the Lamb-Dicke parameter) once the ion has been cooled in the Lamb-Dicke limit.

ACKNOWLEDGMENTS

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