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SCALABLE TRAPPED ION QUANTUM COMPUTATION WITH A PROBABILISTIC ION-PHOTON MAPPING

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We propose a method for scaling trapped ions for large-scale quantum computation and communication based on a probabilistic ion-photon mapping. Deterministic quantum gates between remotely located trapped ions can be achieved through detection of spontaneously-emitted photons, accompanied by the local Coulomb interaction between neighboring ions. We discuss gate speeds and tolerance to experimental noise for different probabilistic entanglement schemes.

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1 Introduction

Trapped ions constitute one of the most promising systems for the implementation of a quantum computer [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16]. It appears unlikely that this system can be scaled by simply adding ions to a single trap, due to the growing complexity of the vibrational mode spectrum and the inefficiency of laser cooling of the collective motion of a large ion crystal to near the ground state. Instead, ion trap multiplexing can be achieved by either shuttling ions through a multiply-connected trap structure [11, 12], or coupling remote ions with a common photon-mediated interaction [1]. For the latter approach, it is generally believed that the ions must be enclosed in a high-finesse optical cavity for deterministic quantum gate operation. This "strong coupling" condition means that the coupling rate g between the ion and the cavity mode should satisfy the requirement

$$g^2 >> \kappa\gamma_s \tag{1}$$

where κ is the cavity decay rate and γ_s is the atomic spontaneous emission rate. Although strong coupling has been achieved for neutral atoms in recent experiments [17, 18, 19], it remains difficult to do the same with trapped ions, although there are significant experimental efforts and achievements [20, 21]. The experimental challenge is that the required small optical cavity volume can interfere with the ion trap operation through uncontrolled charges on the dielectric mirror coatings, and the ion trap electrodes can likewise interfere with the cavity mode through diffraction.

Alternatively, faithful entanglement can be established between remote atoms or ions in a probabilistic fashion even if the strong coupling condition is not satisfied [22, 23, 24, 25, 26, 27, 28, 29]. Here, the interference of photons emitted by two remotely-located ions is detected, and a positive photon count without "which-path" information as to which ion emitted the photon will project the two ions into an entangled state. For this purpose, strong coupling between the ion and the photon is not essential. Photon loss only affects the success probability of a positive photon count, and the fidelity of the entanglement is not reduced. By repeating this kind of entangling protocol several times, one can ultimately get faithful entanglement between the ions. We call this a *probabilistic* source of entanglement as the entangling protocol does not succeed in every trial. With this source of entanglement, one can construct probabilistic quantum gate operations. However, probabilistic gates do not in general lead to scalable quantum computing because of the exponential decrease of the success probability as the number of quantum gates increases.

In this paper, we show that *remote deterministic* quantum gates can be constructed for trapped ions from this source of probabilistic entanglement, when we also allow local deterministic quantum gates between nearby ions. This provides a method to scale up the trapped ion system for large scale quantum computation and communication based on a photon-mediated interaction without requiring strong coupling between the ion and photon.

In this scheme, the quantum register consists of a series of ion pairs (each a logical qubit) that are in different trap regions separated by arbitrary distances. Within each pair, one ion (the logic ion) encodes the quantum information and the second ion (the ancilla) allows the coupling to another ion pair through a probabilistic entanglement protocol. This probabilistic entanglement, combined with conventional local motional gates within each pair, allows for an effective quantum gate between the remote logical qubits. The resulting remote operation is deterministic because the probabilistic entanglement operations can be done off-line, and the failure of an entangling attempt does not destroy the computational quantum state carried by the logical ions.

In Sec. II we will show how to achieve deterministic quantum gates based on the probabilistic entangling operations and how this can seed a scalable trapped ion quantum computation model. We will also show how to use the same setup for implementation of quantum repeaters for achieving scalable long -distance quantum communication. In Sec. III we will compare two probabilistic entangling protocols and discuss the gate speeds and tolerance to noise of each protocol.

2 Scalable quantum computing and networking from probabilistic entangling operations

First, we show how to construct a scalable quantum computation model for trapped ions by a combination of probabilistic remote entangling operations and conventional (motional) local gates. One can easily see that this approach is possible because with the probabilistic entanglement, one can do quantum teleportation, which can teleport the local motional gate to a remote site [30, 31, 32]. Direct teleportation of quantum states back and forth for remote gate operations typically requires two ancillary ions on each node and several Bell measurements, local gate operations, and entanglement preparations [30, 32]. The process actually can be significantly simplified if the purpose is to construct a remote deterministic gate. We need only one ancilla ion and one gate operation on each node, and one time of successful entanglement preparation.

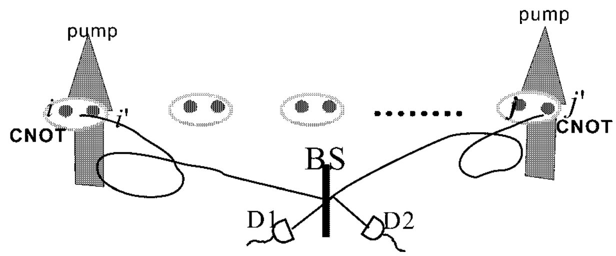

Fig. 1. Schematic illustration of a quantum computation model based on probabilistic photon-mediated entanglement between remote ions. The ancilla ions in different traps are entangled through the probabilistic protocols described in the next section. Deterministic gates on remote ions are constructed from the local motional gates and the probabilistic remote entanglement.

Figure 1 is a schematic of the setup. A series of ion pairs, each in distinct trap regions, are separated by an arbitrary distance. Each qubit is represented by a pair of ions, denoted as i and i' respectively for the logic ion and the ancilla ion. We assume here that the logic ions and the ancilla ions are of different ion species (or isotopes) so that they can be separately addressed for laser manipulation and detection through frequency selection, as has recently been demonstrated in sympathetic cooling experiments [33, 34]. (Of course, they could be the same species ion if one can achieve spatial separate addressing through focused laser beams [6]). In this way, the two ions of a given pair can be tightly confined in a trap with the ability to operate high-fidelity motional quantum gates between them.

To achieve scalability, we should be able to perform deterministic quantum gates between two arbitrary logic ions in different pairs. For this purpose, we assume that each ancilla ion is connected to a single-photon detector, possibly through an optical fiber. To entangle two ancilla ions, say i' and j', we pump both with an appropriate resonant laser beam to excited electronic states. The resulting spontaneously emitted photons from these two ions are directed to single-photon detectors for a Bell-type collective measurement. For particular measurement results, the two ancilla ions i' and j' will be projected into a Bell state, which we denote as $|\Phi_{i'j'}\rangle = (|01\rangle + |10\rangle)/\sqrt{2}$ (see the next section for description of different types of entangling methods). Each entangling operation succeeds with probability p_s (the probability to register the appropriate result), so we need to repeat this operation on average $1/p_s$ times for a final successful confirmation of entanglement, with the total preparation time about t_c/p_s, where t_c is the time for each individual entangling operation. We assume the logic and ancilla ions are sufficiently spectrally resolved so that the probabilistic entangling operation on the ancilla ions does not influence the logic ions, even if this entangling operation fails.

With the assistance of final Bell state $|\Phi_{i'j'}\rangle$, we can achieve remote quantum controlled-NOT (CNOT) gates on the logic ions i and j. We assume that quantum CNOT gates can be realized on the local ions i, i' and j, j' in the same pairs through conventional means relying on the collective motion of the ions [2, 7, 8, 14] (note that all the motional gates work even when the two local ions are of different isotopes or species). These CNOT gates are denoted by $C_{ii'}$ and $C_{jj'}$, where the subscripts refer to the control and target ions. We can achieve the remote CNOT gate C_{ij} on the logic ions i, j through a combination of the gates $C_{ii'}$, $C_{jj'}$ and the Bell state $|\Phi_{i'j'}\rangle$. This can be seen by considering the following identity

$$\begin{aligned} & C_{ii'}C_{jj'}\left(|\Psi\rangle_{ij\ldots}\otimes|\Phi\rangle_{i'j'}\right) \\ = \; & |0+\rangle_{i'j'}\otimes C_{ij}\left(|\Psi\rangle_{ij\ldots}\right)+|0-\rangle_{i'j'}\otimes\sigma_i^z C_{ij}\left(|\Psi\rangle_{ij\ldots}\right) \\ & +|1+\rangle_{i'j'}\otimes\sigma_j^x C_{ij}\left(|\Psi\rangle_{ij\ldots}\right)+|1-\rangle_{i'j'}\otimes\left(-\sigma_i^z\sigma_j^x\right)C_{ij}\left(|\Psi\rangle_{ij\ldots}\right). \end{aligned} \tag{2}$$

where $|\pm\rangle_{j'} = \left(|0\rangle_{j'} \pm |1\rangle_{j'}\right)/\sqrt{2}$, and $|\Psi\rangle_{ij\ldots}$ denotes the computational state, for which the i, j ions may be entangled with other logic ions. The single qubit Pauli operators σ_i^z and σ_j^x act on the corresponding ions i, j. The above identity has been implied previously in different contexts [31, 35, 36, 37], particularly in the discussion of the communication complexity of quantum CNOT gates. The above identity shows that to perform a remote CNOT gate C_{ij} on the logic ions i, j, we can take the following steps:

- Prepare the ancilla ion i' and j' into the EPR state $|\Phi\rangle_{i'j'}$ using a probabilistic entangling protocol. Repeat the protocol until it succeeds.
- Apply the local motional CNOT gates $C_{ii'}$ and $C_{jj'}$ on the ions i, i' and j, j' within the same pairs.
- Measure the ancilla ion i' in the basis $\{|0\rangle_{i'}, |1\rangle_{i'}\}$ and the ancilla ion j' in the basis $\left\{|+\rangle_{j'}, |-\rangle_{j'}\right\}$.
- Apply a single bit rotation $\{I, \sigma_i^z, \sigma_j^x, -\sigma_i^z\sigma_j^x\}$ on ion i and/or j if we get the measurement results $\{0+, 0-, 1+, 1-\}$, respectively.

The resulting remote quantum CNOT gate C_{ij} is deterministic, even though the seeding entangling operations are probabilistic. This is because the probabilistic operation can be repeated off-line until it succeeds. When accompanied by simple local single-bit rotations, this computation model is therefore scalable, with no fundamental limit to the number of ion pairs in different traps. The essential resources are two-ion local motional gates and remote ion-photon probabilistic entangling operations, both of which have been demonstrated [3, 4, 5, 6, 29].

With the same system, we can also realize scalable quantum networks. The basic problem in quantum networking is to transmit quantum states over large distances by overcoming the limit setting by the photon attenuation length. Typically, if one directly sends a single-photon pulse through an optical channel, the efficiency (the probability that the photon is not absorbed) will degrade exponentially with distance due to photon attenuation. One way to overcome this obstacle is based on quantum repeaters [38]. Implementation of quantum repeaters has been proposed in [23] based on the use of atomic ensembles for storage of quantum entanglement, and following this scheme some interesting first-step experiments have been recently reported [39, 40, 41]. The quantum repeater can also be realized in the present context with pairs of ions as discussed above. Figure 2 illustrates schematically the implementation of quantum repeaters with the paired-ion setup.

With the probabilistic entangling protocol, we can generate entanglement between two nodes, say i and k, and also k' and j', each with a communication distance L_0 which is smaller

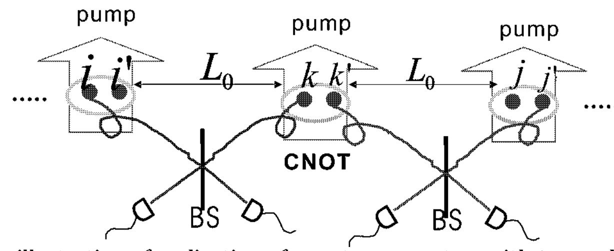

Fig. 2. Schematic illustration of realization of quantum repeaters with trapped ions based on the probabilistic detection-induced remote entanglement and the local Coulomb interaction.

or comparable to the photon attenuation length. The success probability for preparation of each segment of entanglement is given by $p_{sc} = p_s p_c$, where p_s is the inherent success probability of the entangling protocol, and $p_c = e^{-\alpha L_0}$ is the photon attenuation in the channel. These two segments of entanglement can be connected to generate an entangled state between i and j' through a local collective Bell measurement on the two ions k and k' in the same pair. A combination of a motional CNOT gate and individual ion detections achieves the desired collective measurement. The preparation time for each segment of entanglement is $T_{sc} = t_c/p_{sc}$, and the time for establishing entanglement between the next neighboring nodes i and j (with a distance $2L_0$) is simply estimated by $T_2 = 2T_{sc}$ if we sequentially prepare each segment of entanglement. So the time required for establishing entanglement over n segments with a total communication distance of nL_0 is estimated by $T_n = nT_{sc} = ne^{\alpha L_0}(t_c/p_s)$ with the ion-based quantum repeaters. (Here we neglect local motional gate errors and ion detection inefficiency, which are typically small compared with errors from the photon attenuation and the inherent inefficiency of the entangling protocol). This linear scaling of the communication time compares favorably with the exponential scaling law $T_n = e^{n\alpha L_0}(t_c/p_s)$ for the case of direct communication without repeaters.

3 Two types of probabilistic entangling schemes and their corresponding gate speeds

We consider two types of probabilistic entangling protocols [22, 25], denoted as Type I and Type II. Below, we first introduce the basic ideas of these schemes in the context of the $^{111}Cd^{+}$ ion [33]. Then, we compare the merits of each scheme, with particular attention paid to gate speed and tolerance to experimental noise.

3.1 Type I probabilistic entanglement: photon interference

The Type I protocol was first proposed in Ref. [22] and is illustrated in Fig. 3. A weak pump pulse is applied to the atomic transition $|0\rangle \rightarrow |e\rangle$, which excites the atom to the upper state $|e\rangle$ with probability of $p_e << 1$. Spontaneously emitted light from the de-excitation $|e\rangle \rightarrow |1\rangle$ is collected within a cone angle θ (see Fig. 3). The collected light from the two ions is directed to a beamsplitter (BS) for interference and then detected through two single photon detectors D1 and D2. If *one* of the detectors registers a photon, the two ions will be projected onto the state $|\Psi_1\rangle = \left(|01\rangle + e^{i\varphi}|10\rangle\right)/\sqrt{2}$, where the phase φ depends on the difference in path lengths from each ion to the detector. Conditioned on one detector click, there is a probability p_e that both ions spontaneously decay to the state $|1\rangle$ with one of the

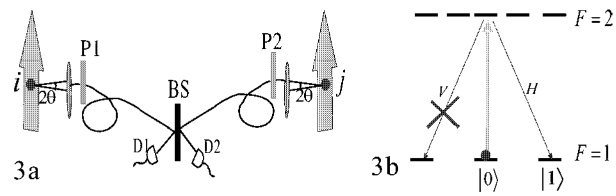

Fig. 3. The type I probabilistic entangling protocol for two remote atomic qubits. Fig. 2a shows the atomic level structure from the ground $F = 1$ states to the upper $F = 2$ states for the $^{111}Cd^{+}$ ion (or any atomic system with nuclear spin 1/2 or 3/2). The V-polarized light in the collection direction is filtered through the two polarizers P1 and P2 so it is not relevant for this protocol. The $|0\rangle$ and $|1\rangle$ states correspond respectively to the Zeeman levels $|m = 0\rangle$ and $(|m = +1\rangle + |m = -1\rangle)/\sqrt{2}$. The excitation probability is required to be low and the scheme succeeds when only one of the detectors fires.

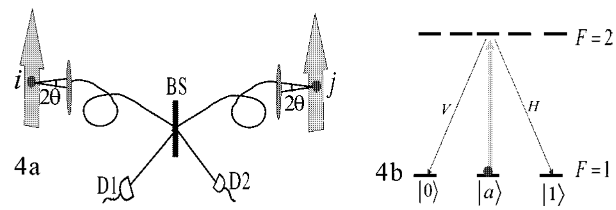

Fig. 4. The type II probabilistic entangling protocol for two remote ions/atoms. The beam splitter (BS) can be replaced by a polarization beam splitter (PBS) together with two polarizers. The $|a\rangle$, $|0\rangle$ and $|1\rangle$ states correspond respectively to the Zeeman levels $|m = 0\rangle$, $(|m = +1\rangle + |m = -1\rangle)/\sqrt{2}$, and $(|m = +1\rangle - |m = -1\rangle)/\sqrt{2}$. One succeeds only when both D1 and D2 register a photon.

accompanying photons not registered by the detectors. The inherent infidelity of this scheme is thus given by p_e.

3.2 Type II probabilistic entanglement: polarization-spin entanglement

The Type II protocol was first proposed in Ref. [25]. Here, there are three relevant atomic levels $|0\rangle, |1\rangle, |a\rangle$ in each of two ions. Figure. 4b shows a configuration of atomic states with an $F = 1$ ground state hyperfine manifold (e.g., within the $S_{1/2}$ ground state of $^{111}Cd^{+}$). The atoms are initially prepared in state $|a\rangle$ and then transferred to states $|0\rangle$ and $|1\rangle$ with unit probability, emitting photons correspondingly either in V or H polarizations along the direction of collection. The joint atom-photon state from two sides can be written as $|\Psi_{ap}\rangle = (|0V\rangle + |1H\rangle) \bigotimes (|0V\rangle + |1H\rangle)/2$. The collected photons traverse a beam splitter (BS) and are then detected with two single-photon detectors. We keep the atomic state if *both detectors* each register a photon, which succeeds only when the accompanying photons before the BS are in the anti-symmetric component $(|HV\rangle - |VH\rangle)/\sqrt{2}$ [42]. With this desired detection event, the state of the two ions is thus projected onto $|\Psi_2\rangle = (|01\rangle - |10\rangle)/\sqrt{2}$. The type II protocol can also be multiplexed to directly prepare multi-ion entangled states if the photon collection efficiency is reasonably high [25].

3.3 Comparison between type I and type II probabilistic entanglement

For the type I protocol, the entanglement success probability, or the probability of detecting a photon is

$$p_I = p_e p_c \eta_d / 2, \tag{3}$$

where η_d is the detection efficiency, and p_c is the photon collection efficiency, with the form of $p_c = 3(1-\cos\theta)/4$ for the setup schematically shown in Fig. 3a (in addition to the fraction of the solid angle $(1-\cos\theta)/2$, the coefficient $3/2$ comes from the space integration of the dipole emission pattern). There is an additional factor of $1/2$ in Eq. (2) since only one polarization is measured after the beam splitter. So the average entangling time for this protocol is $T_I = t_c/p_I$, where t_c is the time for one entangling attempt, which is limited only by the radiative lifetime t_e of the upper level $|e\rangle$, i.e., $t_c > t_e \approx 3\ ns$.

For the type II protocol, the entanglement success probability, or the probability of registering a single photon from each of the detectors is

$$p_{II} = p_c^2 \eta_d^2 / 4. \tag{4}$$

The factor of $1/4$ is the possibility that two photons go to different detectors after the BS. The corresponding average entangling time is $T_{II} = t_c/p_{II}$ (assuming the same t_c for each entangling attempt).

If the above entangling operations are performed *on-line*, the ancilla ions are entangled during the gate operation, which is itself the slowest step of the procedure. In this case, the gate time is then approximately T_I or T_{II} for the type I and type II entangling protocols, respectively. The time T_{II} is larger than T_I if $p_c\eta_d/2 < p_e$, and vice versa. To ensure a reasonable fidelity, the excitation probability p_e is typically about 1%, and in the first experiment with free-space ions [29], $p_c\eta_d \sim 10^{-3}$ from the limited collection solid angle , so $T_{II} \gg T_I$. Nevertheless, the collection efficiency p_c can be significantly enhanced in future experiments (see the discussion below), and we might ultimately expect that $T_I \sim T_{II}$. It is also possible to have $T_{II} < T_I$ if one puts the ions into a fairly good cavity even if it is still far from the strong coupling limit.

The entangling operation can also be done *off-line.* Well before the desired quantum gate operation, the two ancilla ions can be entangled through one of the above probabilistic entangling protocols, and following success this entanglement can be stored for later quantum gate operations. Here, the potentially slow off-line entangling operation is not necessarily a limiting issue for the speed of the subsequent quantum gate operations. The slowest step of the gate is then the detection of the ancilla ions, for which the required time can be estimated by $T_d \sim t_e/p_c\eta_d$. With the quantum jump detection method, we need to register several photons when the ion is at the "bright" level, which takes a time of order $t_e/(p_c\eta_d)$. For instance, with a moderate efficiency $p_c\eta_d \sim 10^{-3}$, $T_d \sim 10\ \mu s$ for the $^{111}Cd^+$ ion.

Now we discuss the tolerance of the type I and type II entanglement protocols to relevant experimental noise. For the type I protocol, the phase φ in the entangled state $|\Psi_1\rangle$ is proportional to $\varphi \sim \Delta k \Delta x$, where Δk is the difference between the wave vectors for the pumping and collected light, Δx is the position fluctuation of the ion from its equilibrium position. The type I protocol thus requires the atom to be confined within the Lamb-Dicke limit, otherwise the residual ion motion will randomize the phase φ and consequently degrade the entanglement fidelity.

The type II entanglement protocol is much less sensitive to fluctuations in the atomic position. This is because the two polarization components carry the same random phase imposed by the instantaneous position of the atom. For this common-mode cancellation of phase fluctuations, the atom must have an approximately fixed (albeit random) position during the emission process. This implies that if the ion is not confined within the Lamb-Dicke limit, the decay rate $1/t_e$ of the upper level $|e\rangle$ need be significantly larger than the frequency of any component of ion motion, which is typically the case. The type-II protocol also exhibits better interferometric stability [27] for the same reason that random phases from the two polarization modes cancel each other. Due to its better noise tolerance, the type II protocol seems more attractive than the type I protocol, although the latter could have a higher success probability. An important seeding step for the type-II protocol has been demonstrated in a recent experiment [29], where entanglement has been directly observed for the first time between a stationary ion spin qubit and a flying photon polarization qubit.

Finally, we consider possibilities for improving the collection efficiency of the photons emitted by the ions. One way is to position the ion in an optical cavity, whereby the effective collection efficiency can be improved by a factor of the cavity finesse compared with the free space case with the same collection solid angle. Even for cavity mirrors with a moderate finesse, the success probability, and thus the gate time, for either protocol above can be significantly improved (particular for the type II protocol, as T_{II} scales quadratically with p_c). Alternatively, the emitted light can be collected by a single-mode optical fiber. This not only filters unwanted spatial modes so that the emission from multiple atoms can be easily mode-matched, but also the collection efficiency could be improved with near-field engineering of the fiber tips. Here, the angular dependence of the emitted photon polarization may cause a reduced amount of entanglement. However, clever linear optical transformations may allow high-fidelity entangled states to be recovered, with some tradeoff in efficiency [43].

In summary, we have proposed a new method to scale up the ion trap system for large scale quantum computing or networking, based on a probabilistic ion-photon mapping. Remarkably, the photon-mediated interaction does not require strong coupling between the ion and the photon. The protocol can also be highly tolerant of relevant experimental imperfections.

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