Fortschr. Phys. **48** (2000) 9−11, 801−810

Ion Trap Quantum Computing with Warm Ions

G. J. MILBURN, S. SCHNEIDER and D. F. V. JAMES*

Centre for Quantum Computer Technology,
The University of Queensland, QLD 4072 Australia.

*University of California, Los Alamos National Laboratory, Los Alamos, NM 87545, USA

Abstract

We describe two schemes to manipulate the electronic qubit states of trapped ions independent of the collective vibrational state of the ions. The first scheme uses an adiabatic method, and thus is intrinsically slow. The second scheme takes the opposite approach and uses fast pulses to produce an effective direct coupling between the electronic qubits. This last scheme enables the simulation of a number of nonlinear quantum systems including systems that exhibit phase transitions, and other semiclassical bifurcations. Quantum tunnelling and entangled states occur in such systems.
PACS: 42.50.Vk,03.67.Lx, 05.50.+q

Ion trapping is currently the most advanced technology for the creation and study of entangled multi-particle states [1, 2, 3]. For many years quantum optics has provided a fertile experimental context to study the entanglement of two [4] and recently three modes [5], however ion trapping provides a clearer path to entangling many more systems either in a single trap or by a network of ion traps with a few ions in each [6]. The objective in an ion trap quantum computer is to create an entangled state between the electronic states of distinct and addressable ions. Recently a partially entangled state of four trapped ions has been achieved [7]. The electronic states of the ions interact weakly or not at all, so the necessary interactions to create entanglement are mediated by the coupling of the electronic states of each ion to a collective vibrational mode. This is the standard ion trap quantum computer architecture first suggested by CIRAC and ZOLLER [8] and requires that the collective vibrational state be carefully controlled. Generally it is required that the particular collective vibrational mode is cooled down to the ground state using laser cooling techniques. Unfortunately the ions do not stay in this ground state but typically undergo some form of heating that results in the irreversible excitation of higher collective vibrational states. Considerable progress has been made in understanding this heating [9] and mitigating it [10]. However it continues to place limitations on current experiments.

A number of workers [11] have proposed alternative schemes to the standard ion-trap QC architecture that do not require the ions to be cooled to the collective ground state. In this paper we will review the two schemes that we have proposed. The first scheme [12] is based on an adiabatic interaction between the collective vibrational modes and the internal electronic states of the ion. While the scheme does succeed in producing an effective interaction between the electronic qubits that is independent of the vibrational state of the ions, the dependence on adiabatic transitions makes it necessarily slow, and thus susceptible to heating that occurs during the adiabatic process itself. The second scheme [13] overcomes this problem by using a sequence of very fast interactions (ideally impulsive) that effectively decouple completely the vibrational and electronic degrees of freedom. The great advantage of this scheme is that in principle it enables a realisation of many nonlinear interactions between the electronic degrees of freedom, including a number of important models that exhibit quantum phase transitions. It thus becomes possible to study entangle-

ment in nonlinear multi-particle systems, outside of the quantum computer paradigm, by directly 'synthesising' interacting multi-particle systems.

1. Adiabatic Scheme

We make use of the fact that although the ions are not necessarily in the phonon mode ground state, they all share the same phonon mode, thus enabling them to interact with each other. In this paper we only consider mitigating the effect of thermal excitation in the vibrational mode, here the centre-of-mass mode, (CM) used for logic operations. However thermal excitation of the other modes is itself a source of decoherence due to the Debye-Waller effect, which causes fluctuations in the effective Rabi frequencies that are used to couple the vibrational and electronic degrees of freedom [1]. If all the other 'spectator' modes are in the ground state this effect can be ignored. We thus assume that except for the centre-of-mass (CM) mode all the other modes have to be in their motional ground state. If the number of ions is large, this constraint can be relaxed to some extent.

We will consider a controlled-rotation (C-ROT) gate between a pair of ions designated control (c) and target (t). To get a controlled not gate (C-NOT) we only need to sandwich the C-ROT gate between single qubit rotations, which do not depend on the vibrational state and thus are not of concern when heating is considered. The gate operation here consists of a conditional sign change which takes place only if both ions are in the excited state. It can be realised by a sequence of four laser pulses.

First a conditional phase shift $\mathcal{S}_t$ is performed between the target qubit and the phonon mode, thereby changing the sign of the wave function only if the phonon mode has an odd excitation and the target ion is in its excited state. This operation can be performed, for example, by applying a detuned laser pulse of well defined duration with the ion at the node of a standing wave of the addressing laser [14]. The next step is to excite an additional phonon into the vibrational mode, conditional on the state of the control ion. This is realised by an adiabatic passage between the excited state $|1\rangle_c$ and some auxiliary state $|2\rangle_c$ of the control ion, which at the same time excites a single phonon, thereby changing an even phonon state to an odd phonon state (and vice-versa). The advantage of using adiabatic passage for this step is that the operation can be carried out independent of the number of phonons. The next step is to perform a second conditional sign change $\mathcal{S}_t$. Finally we disentangle the ion states from the phonon mode by performing the adiabatic passage backwards. As will be shown in detail below, these four pulses produce the desired quantum logic gate assuming that except for the CM mode all the other modes are in their motional ground state. We will now describe those steps in detail. First let us consider the various laser-ion interactions we will need.

To simplify our analysis we will assume that the phonon mode is in a pure state given by the following formula:

$$|\phi\rangle_{\text{ph}} = \sum_n a_n |n\rangle \,, \tag{1}$$

where a_n are a set of unknown complex coefficients and $|n\rangle$ is the Fock state of occupation number n. It will be convenient in what follows to introduce the odd and even parts of this wave function, viz.:

$$\left.\begin{aligned} |\text{even}\rangle_{\text{ph}} &= \sum_n a_{2n} |2n\rangle \,, \\ |\text{odd}\rangle_{\text{ph}} &= \sum_n a_{2n+1} |2n+1\rangle \,. \end{aligned}\right\} \tag{2}$$

We will also use the following notation for phonon states to which a single quantum has been added:

$$\left.\begin{aligned} |\mathrm{odd}'\rangle_{\mathrm{ph}} &= \sum_n a_{2n}\,|2n+1\rangle\,, \\ |\mathrm{even}'\rangle_{\mathrm{ph}} &= \sum_n a_{2n+1}\,|2n+2\rangle\,. \end{aligned}\right\} \tag{3}$$

We should emphasise that our scheme does not require that the CM phonon mode be prepared in either of these states: we are introducing these states for notational convenience only.

The conditional phase change between odd phonon number states and the excited internal state of an ion can be carried out using an effect first considered by D'HELON and MILBURN [14]. They introduced a Hamiltonian for a two-level ion at the node of a detuned classical standing wave. In the limit of large detuning and for interaction times much greater than the vibrational period of the trap, this Hamiltonian for the jth ion in the Lamb-Dicke limit, is

$$H^{(j)} = \hbar \sum_{i=0}^{N-1} a_i^\dagger a_i \chi(\sigma_z^{(j)} + 1/2)\,, \tag{4}$$

where $\sigma_z^{(j)}$ is the population inversion operator for the jth ion, a_i and $a_i^\dagger$ are the annihilation and creation operators of the ith of the N phonon modes, and $\chi = \eta^2\Omega^2/(N\delta)$. Here η is the Lamb-Dicke parameter, Ω is the Rabi frequency for the transitions between the two internal states of the ions, N is the total number of ions and δ the detuning between the laser and the electronic transition. If we choose the duration τ of this interaction to be $\tau = \pi/\chi$, the time evolution is represented by the operator

$$\mathcal{S}_j = \exp\left[-i \sum_{i=0}^{N-1} a_i^\dagger a_i(\sigma_z^{(j)} + 1/2)\,\pi\right]. \tag{5}$$

If we now assume that the CM mode is in an arbitrary vibrational state, but all the other modes are in their respective motional ground state, then this time evolution flips the phase of the ion when the phonon mode is in an odd state and the ion is in its excited state, thus providing us with a conditional phase shift for an ion and the CM phonon mode.

The adiabatic passage [15] which we require for our gate operation can be realised as follows: We use two lasers, traditionally called the pump and the Stokes field. The pump laser is polarised to couple the qubit state $|1\rangle_c$ to some second auxiliary state $|3\rangle_c$ and is detuned by an amount Δ. The Stokes laser couples to the red side band transition $|2\rangle_c\,|n+1\rangle \leftrightarrow |3\rangle_c\,|n\rangle$, with the same detuning Δ. If the population we want to transfer adiabatically is initially in the state $|1\rangle_c\,|n\rangle$, we turn on the Stokes field (i.e. the sideband laser) and then slowly turn on the pump field (i.e. the carrier laser) until both lasers are turned on fully. Then we slowly turn off the Stokes laser: this is the famous "counter-intuitive" pulse sequence used in adiabatic passage techniques [15]. The adiabatic passage has to be performed very slowly. The condition in our scheme is that $T \gg 1/\Omega_{p,n}, 1/\Omega_{S,n}$, where T is the duration of the adiabatic passage and $\Omega_{p,n}$ ($\Omega_{S,n}$) are the effective Rabi frequencies for the pump and the Stokes transition, respectively. Furthermore, in order that the various phonon number states be well resolved, we require that $\Omega_{p,n}, \Omega_{S,n} \ll \omega_{ph}$, the phonon angular frequency. Using the adiabatic passage we can transfer the population from $|1\rangle_c\,|n\rangle$ to $|2\rangle_c\,|n+1\rangle$. To invert the adiabatic passage, we just have to interchange the roles of the pump and the Stokes field. We will denote the adiabatic passage by operators $\mathcal{A}_j^+$

and $\mathcal{A}_j^-$ defined as follows:

$$\mathcal{A}_j^+ : |1\rangle_j |n\rangle \rightarrow |2\rangle_j |n+1\rangle$$
$$\mathcal{A}_j^- : |2\rangle_j |n+1\rangle \rightarrow |1\rangle_j |n\rangle . \tag{6}$$

Putting all those operations together, we can write down the step-by-step states for our gate. We consider the controlled phase shift between the target ion and the phonon mode. For simplicity we will only consider the state that changes.

$$|1\rangle_c |1\rangle_t \{|\text{even}\rangle_{\text{ph}} + |\text{odd}\rangle_{\text{ph}}\} \xrightarrow{\mathcal{S}_t} |1\rangle_c |1\rangle_t \{|\text{even}\rangle_{\text{ph}} - |\text{odd}\rangle_{\text{ph}}\} . \tag{7}$$

The next step is the adiabatic passage as explained above:

$$|1\rangle_c |1\rangle_t \{|\text{even}\rangle_{\text{ph}} - |\text{odd}\rangle_{\text{ph}}\} \xrightarrow{\mathcal{A}_c^+} |2\rangle_c |1\rangle_t \{|\text{odd}'\rangle_{\text{ph}} - |\text{even}'\rangle_{\text{ph}}\} . \tag{8}$$

The next step is the conditional phase flip on the target ion and the CM phonon mode:

$$|2\rangle_c |1\rangle_t \{-|\text{odd}'\rangle_{\text{ph}} - |\text{even}'\rangle_{\text{ph}}\} \xrightarrow{\mathcal{A}_c^-} -|1\rangle_c |1\rangle_t \{|\text{even}\rangle_{\text{ph}} + |\text{odd}\rangle_{\text{ph}}\} . \tag{9}$$

Thus we end up with a controlled rotation gate between the ions c and t. A controlled-NOT (CNOT) gate can be realised by performing $\pi/2$ rotation pulses on the target qubit both before and after this series of operations.

For simplicity, we have analysed these operations under the assumption that the state of the CM phonon mode can be described by an arbitrary pure state. More generally, one must assume that the CM phonon mode is in a mixed state, because it can be entangled with some unknown external quantum system, for example the electromagnetic field causing the heating. Provided we assume that this external system does not become entangled with internal degrees of freedom of the qubits, one can quite easily analyse the gate using a density matrix formalism appropriate for mixed states. Since the adiabatic passage and the conditional phase shift all work for arbitrary CM phonon mode states, our principal result, that gate operations can be performed between arbitrary pairs of qubits, can be shown to be true under these circumstances. However we have implicitly assumed that no heating is taking place during the application of the adiabatic passage. As this process is necessarily slow, this is a difficult assumption to justify in general. It would of course be better if a fast method could be found to effectively decouple the vibrational and electronic motion and it is to such a scheme that we now turn.

2. Fast Pulse Scheme

The interaction Hamiltonian for N ions interacting with the CM vibrational mode can be controlled by using different kinds of Raman laser pulses. A considerable variety of interactions has already been achieved or proposed [1−3]. Consider first the simplest interaction that does not change the vibrational mode of the ions. Each ion is assumed to be driven by a resonant laser field which couples two states, the ground state $|g\rangle$ and an excited state $|e\rangle$. The interaction Hamiltonian is

$$H_I = -\frac{i}{2}\hbar \sum_{i=1}^{N} (\Omega_i \sigma_+^{(i)} - \Omega_i^* \sigma_-^{(i)}) \tag{10}$$

where Ω_i is the effective Rabi frequency at the i'th ion and we have assumed the dipole and rotating wave approximation as usual. The raising and lowering operators for each ion are defined by $\sigma_- = |g\rangle \langle e|$ and $\sigma_+ = |e\rangle \langle g|$. If we now assume that each ion is driven by an identical field and chose the phase appropriately, the interaction may be written as

$$H_I = \hbar \Omega \hat{J}_y \tag{11}$$

where we have used the definition of the collective spin operators,

$$\hat{J}_\alpha = \sum_{i=1}^{N} \sigma_\alpha^{(i)} \tag{12}$$

where $\alpha = x, y, z$ and

$$\sigma_x^{(i)} = \frac{1}{2} \left(\sigma_+^{(i)} + \sigma_-^{(i)}\right), \tag{13}$$

$$\sigma_y^{(i)} = -\frac{i}{2} \left(\sigma_+^{(i)} - \sigma_-^{(i)}\right), \tag{14}$$

$$\sigma_z^{(i)} = \frac{1}{2} \left(|e\rangle_i \langle e| - |g\rangle_i \langle g|\right). \tag{15}$$

The interaction Hamiltonian in Eq. 11 corresponds to a single collective spin of value $j = N/2$ precessing around the $\hat{J}_y$ direction du e to an applied field. By choosing the driving field on each ion to be the same we have imposed a permutation symmetry in the ions reducing the dimension of the Hilbert space from 2^N to $2N + 1$. The eigenstates of $\hat{J}_z$ may be taken as a basis in this reduced Hilbert space. Collective spin models of this kind were considered many decades ago in quantum optics [16] and are sometimes called Dicke models after the early work on superradiance of DICKE [17]. In much of that work however the collective spin underwent an irreversible decay. In the case of an ion trap model however we can neglect such decays due to the long lifetimes of the excited states.

The natural variable to measure is $\hat{J}_z$ as a direct determination of the state of each ion via shelving techniques will give such a measurement. These measurements are highly efficient, approaching ideal projective measurements. The result of the measurement is a binary string which is one of the degenerate eigenstates $\hat{J}_z$. Repeating such measurements it is possible to construct the distribution for $\hat{J}_z$ and corresponding averages. Other components may also be measured by first using a collective rotation of the state of the ions.

We now show how to realise a controlled C-ROT gate for two trapped ions using conditional displacement pulses. By appropriate choice of Raman lasers it is possible to realise the conditional displacement operator for the i'th ion [18]

$$H^{(i)} = -i\hbar(\alpha_i a^\dagger - \alpha_i^* a)\, \sigma_z^{(i)} \tag{16}$$

where a is the annihilation operator for the CM vibrational mode. If the ion is in the excited (ground) state this Hamiltonian displaces the vibrational mode by a complex amplitude α $(-\alpha)$. For a two qubit gate like a C-ROT gate we need a spin-spin interaction. Suppose for example we wish to simulate the interaction of two spins with the Hamiltonian

$$H_{\text{int}} = \hbar \chi \sigma_z^{(1)} \sigma_z^{(2)}. \tag{17}$$

The required pulse sequence is

$$U_{\rm int} = e^{i\kappa_x \hat{X}\sigma_z^{(1)}}\, e^{i\kappa_p \hat{P}\sigma_z^{(2)}}\, e^{-i\kappa_x \hat{X}\sigma_z^{(1)}}\, e^{-i\kappa_p \hat{P}\sigma_z^{(2)}} = e^{-i\chi\sigma_z^{(1)}\sigma_z^{(2)}} \tag{18}$$

where $\hat{X} = (a + a^\dagger)/\sqrt{2},\ \hat{P} = -i(a - a^\dagger)/\sqrt{2}$ and we have used

$$e^{i\kappa_p \hat{P}\hat{J}_z}\, \hat{X}\, e^{-i\kappa_p \hat{P}\hat{J}_z} = \hat{X} + \kappa_p \hat{J}_z\,. \tag{19}$$

The parameters for the displacements are, $\kappa_x = \sqrt{2}\,|\alpha|\,T\sin\theta$ and $\kappa_p = \sqrt{2}\,|\alpha|\,T\cos\theta$ with θ the phase of α , and T the pulse duration. In other words these are the pulse areas for the conditional displacement pulses used when the Hamiltonian Eq. (16) is applied for the appropriate choices of phase in α_i.

This transformation may be used together with single spin rotations to simulate a two spin transformation that is one of the universal two qubit gates for quantum computation. For example the controlled phase shift operation, C-ROT,

$$U_{cp} = e^{-i\pi\,|e\rangle_1\langle e|\otimes|e\rangle_2\langle e|} \tag{20}$$

may be realised with $\chi = \pi$ as

$$U_{cp} = e^{-i\frac{\pi}{2}\sigma_z^{(1)}}\, e^{-i\frac{\pi}{2}\sigma_z^{(2)}}\, U_{\rm int}\,. \tag{21}$$

This transformation does not depend on the vibrational state and so long as it is applied faster than the heating rate of the collective vibrational mode it can describe the effective interaction between two qubits independent of the vibrational mode. However if the pulse is too rapid, there is a possibility of exciting motional sidebands of other collective modes, which would be undesirable. For current ion traps however the pulses considered can be sufficiently rapid to avoid heating without exciting other modes.

The form of Eq. (17) suggests that with an appropriate sequence of pulses it would be possible to simulate an interacting spin chain described by the Ising model. It is easy to verify that this is so. Recently there have been a few suggestions [19, 20] that quantum entanglement plays a role in quantum (zero temperature) phase transitions [21]. The aim is to show a connection between long range order and entanglement. We now show how fast pulse transformations can be used to simulate interesting non linear models with phase transitions.

An interesting model that exhibits a quantum phase transition is the transverse field Ising spin chain [22] which in the simplest case has an interaction Hamiltonian of the form

$$H = -\chi \sum_{i=1}^{N-1} \sigma_z^{(i)}\sigma_z^{(i+1)} - \mu \sum_{i=1}^{N} \sigma_x^{(i)}\,. \tag{22}$$

It is not easy to see how a Hamiltonian of this form could be implemented in an ion trap quantum computer using the method outlined in this paper, which most naturally implements unitary transformations rather than Hamiltonians. However a closely related model described by the unitary operator

$$U = \exp\left(i\chi \sum_{i=1}^{N-1} \sigma_z^{(i)}\sigma_z^{(i+1)}\right) \exp\left(i\mu \sum_{i=1}^{N} \sigma_x^{(i)}\right) \tag{23}$$

could easily be implemented. The second factor is simply a sequence of single qubit rotations on each ion. The explicit pulse sequence for the first factor is,

$$\prod_{i=1}^{N-1} e^{i\kappa_x \hat{X}\sigma_z^{(i+1)}}\, e^{i\kappa_p \hat{P}\sigma_z^{(i)}}\, e^{-i\kappa_x \hat{X}\sigma_z^{(i+1)}}\, e^{-i\kappa_p \hat{P}\sigma_z^{(i)}} \,. \tag{24}$$

In fact the unitary operator in Eq. (23), in the language of quantum circuits, represents N-qubits that are first subjected to a set of Hadamard single qubit rotations and then successive nearest neighbour two qubit conditional phase shifts which could be implemented using the conditional displacements described in this paper. A related model, with more symmetry (but no spatial extent) would be

$$U = \exp\left(2i\chi \sum_{i,j=1}^{N} \sigma_z^{(i)}\sigma_z^{(j)}\right) \exp\left(i\mu \sum_{i=1}^{N} \sigma_x^{(i)}\right) \tag{25}$$

$$= \exp\left(i\chi \hat{J}_z^2\right) \exp\left(-iN\chi\right) \exp\left(i\mu \hat{J}_x\right), \tag{26}$$

where we have written the required unitary transformations in terms of the collective operators. This is the nonlinear top model that was introduced by HAAKE [23, 24] as a quantum chaotic prototype. We now discuss how this can be implemented.

The factor containing the linear rotation in Eq. (26) can easily be implemented using the method described in the first paragraph of this section. To implement the nonlinear part we proceed as follows. In the case of N ions with each driven by identical Raman lasers, the total Hamiltonian is

$$H = -i\hbar(\alpha a^\dagger - \alpha^* a)\, \hat{J}_z \,. \tag{27}$$

By an appropriate choice of Raman laser pulse phases for conditional displacements we can then implement the following sequence of unitary transformations

$$U_{\mathrm{NL}} = e^{i\kappa_x \hat{X}\hat{J}_z}\, e^{i\kappa_p \hat{P}\hat{J}_z}\, e^{-i\kappa_x \hat{X}\hat{J}_z}\, e^{i\kappa_p \hat{P}\hat{J}_z} \tag{28}$$

where $\kappa_x = \sqrt{2}\,|\alpha|\, T \sin\theta$ and $\kappa_p = \sqrt{2}\,|\alpha|\, T \cos\theta$ with θ the phase of α, and T the pulse duration. It is easy to see that

$$U_{\mathrm{NL}} = e^{-i\phi \hat{J}_z^2} \tag{29}$$

where $\phi = \kappa_x \kappa_p$. This is the unitary transformation generated by a nonlinear top Hamiltonian describing precession around the $\hat{J}_z$ axis at a rate dependant on the z component of angular momentum. Such nonlinear tops have appeared in collective nuclear models [25] and form the basis of a well known quantum chaotic system [23].

It should be noted that the transformation in Eq. (29) contains no operators that act on the vibrational state. It is thus completely independent of the vibrational state and it does not matter if the vibrational state is cooled to the ground state or not. A similar result using a different kind of time dependant Hamiltonian has also been presented by SØRENSEN and MØLMER [26]. In a recent work these same authors have shown the connection between their approach and the method used in this paper [27]. However Eq. (29) only holds if the heating of the vibrational mode can be neglected over the time it takes to apply the conditional displacement operators. We discuss below what this implies for current experiments.

In itself the unitary transformation in Eq. (29) can generate interesting states. For example if we begin with all the ions in the ground state so that the collective spin state is initially $|j,-j\rangle_z$ and apply laser pulses to each electronic transition according to the Hamiltonian in Eq. (11) for a time T such that $\Omega T = \pi/2$ the collective spin state is just the $\hat{J}_x$ eigenstate $|j,-j\rangle_x$. If we now apply the nonlinear unitary transformation in Eq. (29) so that $\phi = \pi/2$ we find that the system evolves to the highly entangled state

$$|+\rangle = \frac{1}{\sqrt{2}}\left(e^{-i\pi/4}\,|j,-j\rangle_x + (-1)^j\, e^{i\pi/4}\,|j,j\rangle_x\right). \tag{30}$$

Such states have been considered by BOLLINGER et al. [28] in the context of high precision frequency measurements, and also by SANDERS [29]. They exhibit interference fringes for measurements of $\hat{J}_z$. As noted above a measurement of $\hat{J}_z$ is easily made simply by reading out the state of each ion using highly efficient fluorescence shelving techniques.

Let us now consider the full nonlinear top unitary map Eq. (26). A corresponding classical map can be defined as a two dimensional map of points on a sphere of radius j [24]. A rich fixed point structure and bifurcation sequence results as χ and μ are varied. The classical bifurcations have a quantum analogue in the structure of the eigenstates of the quantum unitary operator [29]. For the purposes of this paper we will discuss one such bifurcation which can be used to generate a cat state. We will fix $\mu = \pi/2$. For small values of χ the points $X = \pm 1, Y = Z = 0$ are stable(elliptical) fixed points of the classical map. In the quantum case these correspond to two eigenstates of the unitary operator which are very well approximated by the extremal eigenstates, $|j,\pm j\rangle_x$, of $\hat{J}_x$. These classical fixed points remain stable until $\chi = 2$ when they become unstable and two new stable fixed pints are born in each of the hemispheres $X > 0$, $X < 0$. We will restrict ourselves to the $X > 0$ hemisphere. In this hemisphere the two fixed points are located on the line $Y = -Z$. Corresponding to these two new stable fixed points are two, nearly degenerate eigenstates, $|\phi\rangle_\pm$, of the unitary operator. A state localised on one of the fixed points will exhibit tunnelling to the other fixed point on the line $Y = -Z$ [24]. This would be a remarkable demonstration of quantum nonlinear dynamics arising through a classical bifurcation.

Initial states of the kicked top can be easily be prepared as coherent angular momentum states by appropriate linear rotations. In the basis of orthonormal $\hat{J}_z$ and $\hat{\mathbf{J}}^2$ eigenstates, the spin coherent states can be written as a rotation of the collective ground state [23, 30] through the spherical polar angles (θ, ϕ),

$$|\gamma\rangle = \exp\left[i\theta(\hat{J}_x \sin\phi - \hat{J}_y \cos\phi))\right]|j,-j\rangle_z \tag{31}$$

where $\gamma = e^{i\Phi}\tan\left(\frac{\theta}{2}\right)$. This can be achieved by identical, appropriately phased pulses on each ion. Initial states localised in either the regular or chaotic regions of the classical phase space may thus be easily prepared.

An experimental demonstration would require measuring the mean value of $\hat{J}_z$ as a function of the number of times the unitary operator is applied. As discussed above this is easily done with high efficiency using fluorescence shelving techniques. If we were to stop the dynamics at half the tunnelling period, the resulting state would be a superposition of two states of localised angular momentum. Such a state would be entangled with respect to the underlying electronic levels of each ion, as in Eq. (29).

We have proposed two schemes for coherent manipulation of the electronic qubits in an ion trap quantum computer that do not require the ions be cooled to the ground state of the collective motion. The second method is based on conditional displacements of a collective vibrational mode and can be used to simulate a variety of nonlinear spin models using a linear ion trap and provide a new path to the creation of highly entangled states via quantum phase transitions. However both schemes require that the heating of the collective

vibrational mode is negligible over the time of the application of the adiabatic pulses in the first scheme, or the Raman conditional displacement pulses in the second scheme. It does not matter that the ion heats up between pulses. If the pulses were applied for times comparable to the heating times the pulse sequences described above would not be defined by a product of unitary transformations but rather by the completely positive maps which include the unitary part as well as the nonunitary heating part. Such maps provide a means to test various thermodynamic limits of nonlinear spin models with finite temperature and will be discussed in a future paper. In current experiments the heating time varies considerably and depends on trap design, and can be as long as 190 ms. [9, 10]. In experiments, the sequence of conditional displacements would need to be applied on time scales of less than about 100 ms. This is easily achievable using Raman pulses. We thus conclude that simple collective and interacting spin models with a few spins are within reach of current ion trap quantum computer experiments.

Acknowledgments

We would like to thank Ross Mckenzie and David Wineland for useful discussions. GJM would also like to thank The Isaac Newton Institute, Cambridge, for the support of a visit during which some of this work was done.

References

[1] D. J. Wineland, C. Monroe, W. M. Itano, D. Leibfried, B. E. King, and D. M. Meekhof, *Experimental issues in coherent quantum-state manipulation of trapped atomic ion*, Journal of Research of the National Institute of Standards and Technology **103**, 259 (1998).
[2] R. J. Hughes, D. F. V. James, J. J. Gomez, M. S. Gulley, M. H. Holzscheiter, P. G. Kwiat, S. K. Lamoreaux, C. G. Peterson, V. D. Sandberg, M. M. Schauer, C. M. Simmons, C. E. Thorburn, D. Tupa, P. Z. Wang, and A. G. White, Fortschritte der Physik **46**, 329 (1998).
[3] Mark S. Gulley, Andrew White, and Daniel F. V. James, *A Raman approach to quantum logic in Calcium-like ion*, submitted to J. Opt. Soc. Amer. B (1999).
[4] A. Aspect, P. Grangier, and G. Roger, Phys. Rev. Lett. **47**, 460 (1981); Z. Y. Ou and L. Mandel, Phys. Rev. Lett., **61**, (1988); P. G. Kwiat, K. Mattle, H. Weinfurter, A. Zeilinger, A. V. Sergienko, and Y. H. Shih, Phys. Rev. Lett, **75**, 4337 (1995); W. Tittel, J. Brendel, N. Gisin, and H. Zbinden, Phys. Rev. Lett. **81**, 3563 (1998).
[5] D. Bouwmeester, Jian-Wei Pan, Matthew Daniell, Harald Weinfurter, and A. Zeilinger, Phys. Rev. Lett. **82**, 1345(1999).
[6] J. I. Cirac, P. Zoller, H. J. Kimble, and H. Mabuchi, Phys. Rev. Lett. **78**,3221 (1997).
[7] C. A. Sackett et al., Nature **404**, 256 (2000).
[8] J. I. Cirac and P. Zoller, Phys. Rev. Lett. **74**, 4091 (1995).
[9] Q. A. Turchette, D. Kielpinski, B. E. King, D. Leibfried, D. M. Meekhof, C. J. Myatt, M. A. Rowe, C. A. Sackett, C. S. Wood, W. M. Itano, C. Monroe, and D. J. Wineland, quant-ph/0002040,(2000).
[10] Ch. Roos, Th. Zeiger, H. Rohde, H. C. Nägerl, J. Eschner, D. Leibfried, F. Schmidt-Kaler, and R. Blatt, Phys. Rev. Lett. **83**, 4713 (1999).
[11] J. F. Poyatos, J. I. Cirac and P. Zoller, Phys. Rev. Lett. **81**, 1322 (1998); A. Sørensen and K. Mølmer, Phys. Rev. **81**, 1322 (1999); S. Schneider, D. V. James and G. J. Milburn, J. Mod. Optics **47**, 499 (1999); G. J. Milburn, submitted to Phys, Rev A (quant-ph/9908037), (2000)
[12] S. Schneider, D. V. James and G. J. Milburn, J. Mod. Optics **47**,499 (1999)
[13] G. J. Milburn, submitted to Phys, Rev A (quant-ph/9908037), (2000).
[14] C. D'Helon and G. J. Milburn, Phys. Rev. A **54**, 5141 (1996).
[15] K. Bergmann and B. W. Shore, *Coherent Population Transfer*, in: *Molecular Dynamics and Spectroscopy by Stimulated Emission Pumping*, H.-L. Dai and R. W. Field, eds. (World Scientific, Singapore, 1995).

[16] P. D. DRUMMOND, Phys. Rev. A **22**, 1179 (1980), and references therein.
[17] R. H. DICKE, Phys. Rev. **93**, 99 (1954).
[18] C. MONROE, D. M. MEEKHOF, B. E. KING, and D. J. WINELAND, Science, **272**, 1131 (1996).
[19] DORIT AHARONOV, *A Quantum to Classical Phase Transition in Noisy Quantum Computers*, quant-ph/9910081(1999).
[20] W. K. WOOTTERS, *Entangled chains*, quant-ph/000114 (2000).
[21] S. L. SONDHI, S. M. GIRVIN, J. P. CARINI, and D. SHAHAR, Rev. Mod. Phys. **69**, 315 (1997).
[22] B. M. MCCOY and T. T. WU, *The two dimensional Ising model*, (Harvard University, Cambridge, 1973).
[23] F. HAAKE, M. KUŚ and R. SCHARF, Z. Phys. B **65**, 381 (1987).
[24] B. C. SANDERS and G. J. MILBURN, Z. Phys. B **77**, 497 (1989).
[25] P. RING and P. SCHUCH, *The Nuclear Many Body Problem*, 110−112, (Springer, New York, 1980).
[26] A. SØRENSEN and K. MØLMER, Phys. Rev. Lett. **81**, 1322 (1999)
[27] A. SØRENSEN and K. MØLMER, *Entanglement and quantum computation with ions in thermal motion*, quant-ph/0002024 (2000).
[28] J. J. BOLLINGER, WAYNE M. ITANO, D. J. WINELAND and D. J. HEINZEN, Phys. Rev. A **54**, 4649 (1996).
[29] B. C. SANDERS, Phys. Rev. A **40** 2417, (1989).
[30] F. T. ARRECHI, E. COURTENS, R. GILMORE and H. THOMAS, Phys. Rev A, **6**, 2211 (1972).