Coaxial Resonators with Helical Inner Conductor*

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***Summary*—The use of coaxial or reentrant resonators is practical [a]t frequencies down through the high-frequency band and even [lo]wer. By introducing a helical inner conductor, a *Q* of several [th]ousand can be achieved in relatively small volume. Design equa[ti]ons are simple and are the basis for an alignment chart. The un[lo]aded *Q* is equal to 50 times shield diameter in inches times square [ro]ot of resonance frequency in megacycles. Shield length is about 30 [pe]r cent greater than its diameter. Experimental results confirm the [ac]curacy of the predicted *Q*. The method is also applicable to min[ia]turized UHF resonators, and to the design of high-*Q* radio-fre[q]uency inductors and LC resonators. Additional formulas and a [ch]art relate frequency, *Q*, volume, voltage gradient and power [ra]ting.**

List of Symbols

b = axial length of coil, inches.
B = inside length of shield, inches.
C = capacitance, micromicrofarads per axial inch.
D = inside diameter of shield, inches.
d = mean diameter of turns, inches.
d_0 = diameter of conductor, inches.
f_0 = resonance frequency, megacycles.
I_{sc} = current at connection of coil to shield.
k = dissipation factor at surface of shield, watts per square inch.
L = inductance, microhenries per axial inch.
N = total number of turns of winding.
n = turns per inch.
P_c = power converted into heat in the resonator, watts.
P_m = maximum power available from a generator into a load, watts.
P_s = power rating of zero- or infinite-impedance generator, watts.
Q_d = doubly-loaded Q.
Q_s = singly-loaded Q.
Q_u = unloaded Q.
R_c = resistance due to coil conductor, ohms per axial inch.
R_s = resistance appearing in a coil due to the losses in the shield, ohms per axial inch.
V_{oc} = voltage at open-circuited end of coil.
(vol) = inside volume of shield, cubic inches.
Z_0 = characteristic impedance, ohms.
α = attenuation constant, nepers per unit length.
β = phase constant, radians per unit length.
δ = skin depth, inch.
$\tau = 1/n$ = pitch of winding, inches.
ϕ = proximity factor.

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Introduction

Resonators of practical size with Q in excess of 1000 can be built for the hf and VHF ranges. They resemble the familiar coaxial-line quarter-wave resonator except that the inner conductor is wound in a helix. The technique can be extended even through the UHF range and beyond, where subminiature helical resonators can be built in smaller sizes than conventional ones when Q values between several hundred and 1000 suffice.

As an example of the saving of space and the much better shape factor, a helical resonator for 10 mc with unloaded $Q = 1000$ is about six inches in diameter by eight inches in length. In contrast, a TEM-mode coaxial-line resonator would be 25 feet in length by three inches in diameter. As an alternative solution, it might be possible to build a lumped (electrically short) inductor and capacitor tuned circuit with $Q = 1000$. However, it would tax one's ingenuity to fabricate it in a six- by eight-inch cylindrical shield.

Again, consider a subminiature resonator with $Q = 200$ for operation at 2000 mc. The helical type would be about one-tenth inch in diameter by one-eighth inch in length. A TEM-mode unit would be 1.5 inches in length by 0.05 inch in diameter.

Fig. 1 shows a series of resonator coils and their shield. The caption gives the resonance frequency and measured unloaded Q of each coil when mounted in the shield.

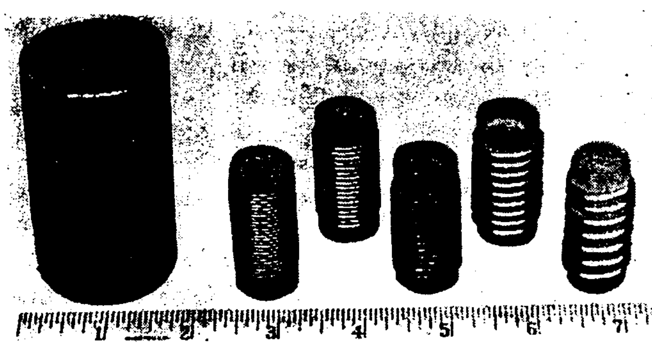

Fig. 1—Group of helical resonators. Shield inside diameter = 1.63 inches. Coils, from left to right: $f_0 = 55$ mc, $Q_u = 600$; $f_0 = 78$ mc, $Q_u = 720$; $f_0 = 101$ mc, $Q_u = 840$; $f_0 = 145$ mc, $Q_u = 880$; $f_0 = 215$ mc, $Q_u = 1000$.

The helical resonator (see Fig. 2) consists of a coil within a shield, one end of the coil being solidly connected to the shield. The other end is open-circuited, except possibly for a trimming capacitor. The resonator thus resembles an ordinary radio-frequency tuned circuit with the omission of the tuning capacitor. How-

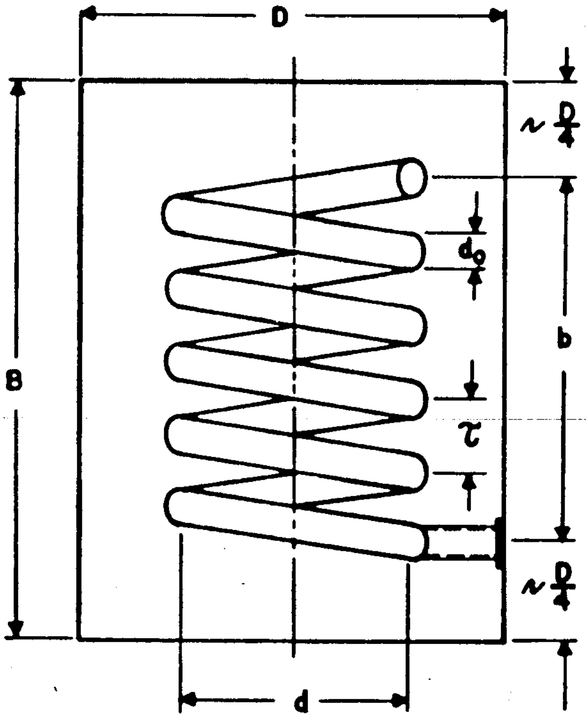

Fig. 2—Outline sketch of resonator.

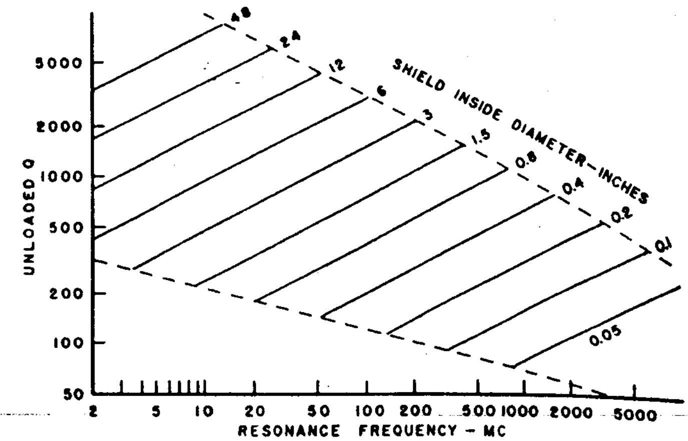

Fig. 3—Helical resonator unloaded Q.

ever, instead of being a lumped-constant device, its operation can be described in terms of its distributed inductance, capacitance, and resistance. The clearance at the top between the end of the helix and the shield is required to prevent voltage flashover, and the clearance at the bottom allows for passage of the magnetic field, thus reducing losses in the conductors. Resonance frequency and Q are about the same when the top and/or bottom are open as when they are closed. Probe, loop or aperture coupling can be used for the input and output circuits.

An idea of the size of the helical resonator for typical unloaded Q and resonance frequency can be gained from Fig. 3, which is plotted from (1), (5) and (7). The optimum range lies between the upper and lower dashed lines. At higher Q_u and f_0 a conventional coaxial resonator is frequently more desirable than the helical type. At lower Q_u and f_0 than the indicated range, a lumped LC circuit is often to be preferred.

A shielded inductor below its quarter-wave resonance frequency also has Q predicted by (1). Then a lumped LC resonant circuit has like Q, provided losses in the capacitor and connecting leads are reduced to a negligible value.

A similar problem has been developed by a somewhat different method than that used herein.[1] A report[2] of extensive experimental work on the resistance and Q of unshielded single-layer solenoids includes tables and charts. This leads to a simple formula for the Q of an inductor similar to that given here for a resonator. A convenient compilation[3] of formulas for resonant lines in general has been made.

Design Chart

The alignment chart of Fig. 4 gives practical design and performance information. Due to the wide range of the parameters covered by the chart, and the latitude of possibilities in actual design and contruction of resonators, it is easily possible in specific cases to have variations of ± 10 per cent. The chart is drawn for the approximate optimum range of the parameters d/D, b/d and d_0/τ.

The equations and conditions for the scales of the chart are developed in later sections hereof. Summarizing:

Unloaded Q of a resonator consisting of a single-layer coil of copper conductor on a low-loss form, and enclosed in a copper shield:

$$\begin{aligned} Q_u &= 50Df_0^{1/2} \\ 0.45 &< d/D < 0.6 \\ b/d &> 1.0 \\ 0.4 &< d_0/\tau < 0.6 \text{ at } b/d = 1.5 \\ 0.5 &< d_0/\tau < 0.7 \text{ at } b/d = 4.0 \\ d_0 &> 5\delta \text{ where } \delta = \text{skin depth}. \end{aligned} \tag{1}$$

Total number of turns:

$$\begin{aligned} N &= 1900/(f_0D) \text{ turns} \\ d/D &= 0.55 \\ b/d &> 1.0. \end{aligned} \tag{2}$$

[1] W. Sichak, "Coaxial line with helical inner conductor," Proc. IRE, vol. 42, pp. 1315–1319; August, 1954.

[2] R. G. Medhurst, "HF resistance and self capacitance of single-layer solenoids," *Wireless Engineer*, vol. 24, pp. 35–43, 80–92, 185, 281; February, March, June and September, 1947.

[3] "Reference Data for Radio Engineers," fourth ed., ITT Corp., New York, N. Y., pp. 574–582, 600–603; 1956.

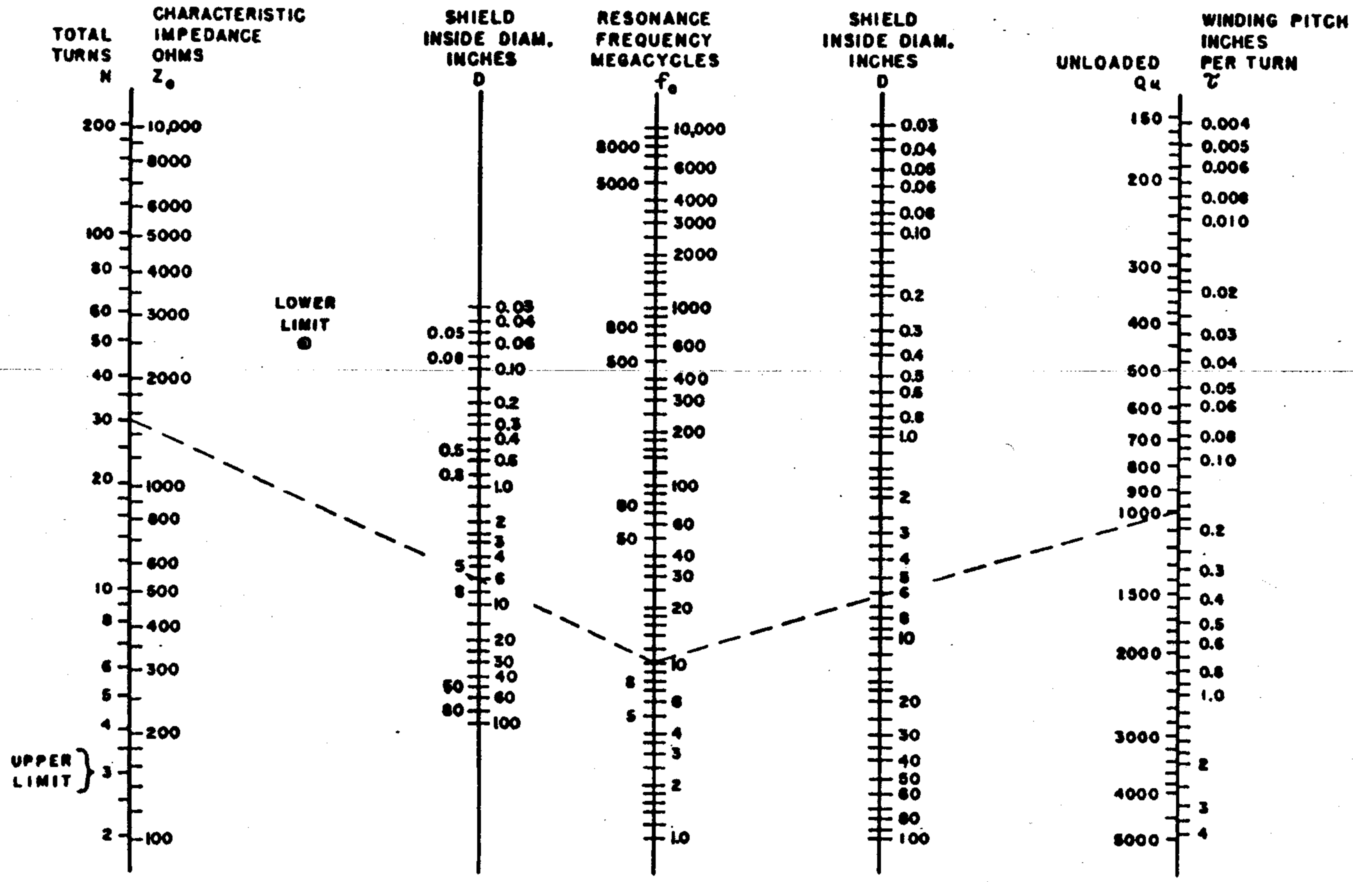

Fig. 4—Design chart for quarter-wave helical resonators.

'itch of winding and characteristic impedance:

$$\tau = 1/n = (f_0D^2)/2300 \text{ inches per turn} \quad (3)$$

$$Z_0 = 98{,}000/(f_0D) \text{ ohms}$$

$$d/D = 0.55$$

$$b/d = 1.5. \quad (4)$$

;eneral conditions for all charts are:

$$B \approx (b + D/2)$$

$$\tau < d/2.$$

The usual precautions for fabricating high-Q coils ıust be observed. A protective silver plating on the ırface of the copper wire and shield is recommended. A lver-clad or solid silver conductor increases the Q ›out 3 per cent over that for silver-plated copper. A nned conductor can be substituted for a silver-plated ıe up to about 100 mc without seriously affecting the . The shield should not have a seam parallel to the ;is; any seam must be effectively soldered for a low- sistance joint. It is desirable to carry the lower end of ıe coil over to the side of the shield as directly as pos- ble. A minimum amount of dielectric material should : used in the coil form, and it should be eliminated en- rely when possible.

Region of Usefulness

The region where straight coaxial resonators are preferable is separated from that where the use of a helical inner conductor is desirable by a rather broad transition zone. A resonator with about three turns lies within the transition zone. On the chart of Fig. 3 the upper dashed line represents this boundary. In the region near and above the line a straight coaxial resonator should be considered, while below it the helical one is usually preferred. The choice is affected by the shape factor. A coaxial resonator is long and relatively small in diameter. In the helical resonator the length is not much greater than its diameter.

This number, $N=3$, can be derived from either of two basic concepts. The first is the locus of points where equal unloaded Q vs frequency is obtained with the two types of resonators having equal volumes. For the straight inner conductor type[4] the theoretical Q is derated about 10 per cent for a practical working value. This is compared with (22) for the helical type. The second basic concept is the limitation that $\tau < d/2$, or the pitch of the helix be less than its radius (other-

[4] F. E. Terman, "Radio Engineers Handbook," McGraw-Hill Book Co., Inc., New York, N. Y., 1st ed., p. 192; 1943.

wise it almost ceases to be helical). When $b/d=1.5$, $N=bn=b/\tau>2b/d=3$ turns. The result in either case is

$$Q_u f_0^{1/2} < 32{,}000 \text{ or } N > 3 \text{ turns}, \tag{5}$$

as the more desirable criterion for the helical resonator.

The demarkation between the relative desirability of a helical resonator vs a lumped-constant tuned circuit is also broad. The lower dashed line on Fig. 3 and the point marked "lower limit" on Fig. 4 are drawn for the condition that the diameter of conductor be greater than five times the skin depth (otherwise the helical-resonator Q will be lower than predicted by the charts). If the alignment on Fig. 4 lies above the "lower limit" point an LC circuit may be preferable. The skin depth is[5]

$$\delta = 2.60 f^{-1/2} \times 10^{-3} \text{ inch}. \tag{6}$$

Utilizing (1) and (3) there results

$$Q_u f_0^{1/4} > 385 \text{ or } f_0^{3/4} D > 7.75, \tag{7}$$

as the condition for the helical resonator being more desirable than the LC circuit. Below this region, for a given volume and Q it becomes advantageous to use fewer turns of larger diameter conductor along with an added capacitor for resonance.

Example

Design a resonator with $Q_u=1000$ at resonance frequency $f_0=10$ mc. By the charts and formulas, $D=6.3$ inches and $N=30$ turns, $\tau=0.174$ inch per turn and $Z_0=1550$ ohms, provided $d/D=0.55$ and $b/d=1.5$, approximately. Let $n=6$ turns per inch in round numbers. Other dimensions are $d=3.5$ inches, $b=5$ inches, $B=5+(6.3)/2=8.2$ inches. The conductor size can be No. 14 to No. 10 B&S gauge ($d_0=0.064$ to 0.102 inch) corresponding to d_0/τ ranging from about 0.4 to 0.6.

The power rating of the resonator can be estimated by use of (32) or (33). Suppose a matched generator is used and a doubly-loaded Q of 100. Assuming a dissipation factor $k=0.4$ watt per square inch, then the generator can have an available power of $P_m=460$ watts. Of this, 20 per cent, or $P_c=92$ watts, is dissipated in the cavity.

If the generator is zero- or infinite-impedance and the singly-loaded Q is 100, the loss in the resonator is 10 per cent. A generator rated at 920 watts can be accommodated.

Experimental Results

The properties of resonators designed in accordance with the chart have been checked experimentally in a considerable variety of models. Fig. 5 shows unloaded Q vs quarter-wave resonance frequency for 26 resonators in 4 size groups. Mechanical tolerance in construction resulted in deviations of up to 5 per cent from the standardized proportions. There was no particular uniformity of conductor material or surface condition. Silver-clad copper, tinned copper, bare copper, and enameled copper were used according to ready availability in the appropriate diameter. The shields consisted of lengths of commercial copper tubing, with no special attention given to surface condition except for the smallest sizes, which were silver plated.

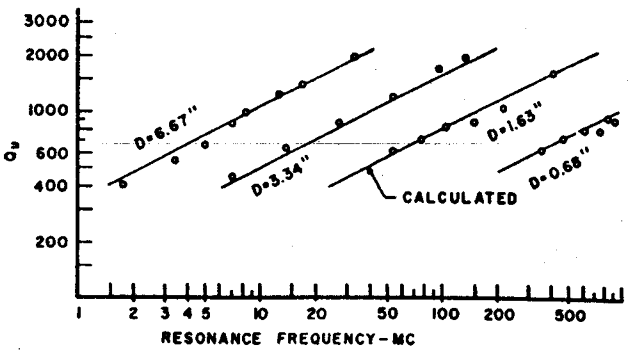

Fig. 5—Comparison of measured and computed Q.

Nearly all of the helices were wound on grooved tubular forms of Rexolite 1422 having wall thicknesses of one-sixteenth to one-quarter inch, depending on the depth of groove required. The helices consisting of only a few turns of heavy conductor were self supporting. In one case, a dielectric-supported helix (shown on the graph in Fig. 5 at $f_0=33$ mc, $Q_u=2000$) was duplicated without the form for comparison. The self-supported helix exhibited about 10 per cent higher Q and 3 per cent higher resonance frequency.

Tests were made to check the predicted optimum d/D ratio and the accuracy of the Q indicated by (22). The plot in Fig. 6 shows typical data, in this case for $b/d=1.5$ and $d_0/\tau=0.5$. Ordinates give the ratio of the measured Q to the predicted value. The curve confirms that the maximum Q occurs when d/D is in the vicinity of 0.55. The maximum value of measured Q exceeds the calculated value by about 8 per cent. This is within the normal expected tolerance of predicted Q, due to variations of materials and construction.

The Q calculated by (1) is based only on the frequency at which the helix is used, and is not restricted to the special case of quarter-wave resonance. The prediction should be valid also for the helix as a lumped inductor, where the electrical length is quite small. To test this, a typical helix was tuned downward in frequency by external capcitance. Some difficulty was experienced in finding capacitors having a Q much higher than that of the helix. The desired tuning range with reasonable losses was covered by a composite array of transmitting-type vacuum capacitors and handmade parallel-plate air dielectric trimmers. The Q vs frequency is shown in Fig. 7.

[5] "Reference Data for Radio Engineers," *op. cit.*, p. 129.

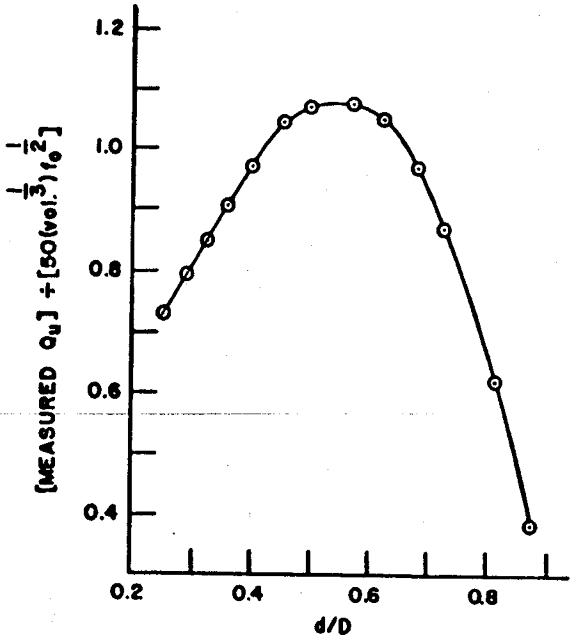

Fig. 6—Test for optimum coil-to-shield diameter ratio.

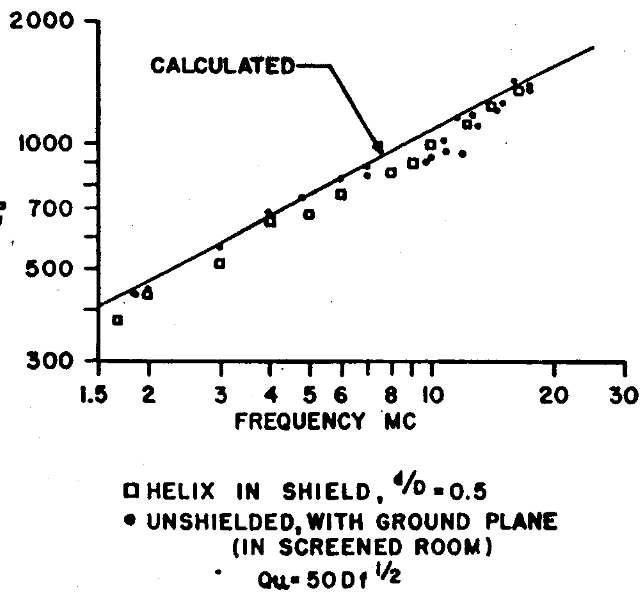

Fig. 7—Measured Q of helix below quarter wave (capacitance tuned).

The data seem to confirm the predicted square-root-of-frequency variation in Q. The scattering of points is accounted for by losses in the external capacitors and associated leads. The uppermost points are for quarter-wave resonance with no capacitor. Those down to 12 mc were obtained by using closely-spaced copper tabs as capacitors. Below 12 mc the effect of introducing the larger, lower-Q capacitors is evident.

Theory

This elementary theory of the quarter-wave helical resonator makes no pretense of being rigorous. Rather, it is directed merely toward developing simple equations and charts for the satisfactory design of resonators.

It has been found to be practical to compute the inductance and capacitance per unit length of the resonator coils as if they were very long. The inductance of a long solenoid[6] as modified by the effect of the shield[7] is

$$L = 0.025n^2d^2[1 - (d/D)^2]\ \mu\text{h per axial inch.} \quad (8)$$

On the basis of measurements of the resonance frequency and characteristic impedance of various resonators, it appears that the effective capacitance is somewhat greater than that of two simple coaxial cylinders. Also, the electrical length of the helix is 5 to 7 per cent less than quarter wavelength. These are attributed in part to the self capacitance of the coil and the fringing field at the top of the coil. An empirical value is:

$$C = 0.75/\log_{10}(D/d)\ \mu\mu\text{f per axial inch.} \quad (9)$$

The velocity, and hence the axial length, of the resonator helix are:

$$v = f_0\lambda = 1000\,(LC)^{-1/2}\ \text{inches per microsecond,} \quad (10)$$

$$b = 0.94\lambda/4 = 235f_0^{-1}(LC)^{-1/2}\ \text{inches.} \quad (11)$$

Substituting (8) and (9) in (11):

$$1/\tau = n = \frac{1720}{f_0D^2(b/d)(d/D)^2}\left[\frac{\log_{10}(D/d)}{1-(d/D)^2}\right]^{1/2}\ \text{turns per inch,} \quad (12)$$

$$N = nb = \frac{1720}{f_0D(d/D)}\left[\frac{\log_{10}(D/d)}{1-(d/D)^2}\right]^{1/2}\ \text{turns.} \quad (13)$$

The characteristic impedance is

$$Z_0 = 1000(L/C)^{1/2} = 235{,}000(bf_0C)^{-1}$$

$$= \frac{0.31 \times 10^6 \log_{10}(D/d)}{f_0D(b/d)(d/D)}$$

$$= 183nd\{[1 - (d/D)^2]\log_{10}(D/d)\}^{1/2}\ \text{ohms.} \quad (14)$$

Unloaded Q

The losses consist of conductor losses[8,9] in the winding and shield, and dielectric losses. The latter are quite small in coils of good design, but are not readily computed. The references noted give formulas for the ac resistance of a straight conductor, the proximity effect due to current in nearby turns and resistance due to currents in the shield. For copper helix and nonmagnetic shield

[6] "Radio Instruments and Measurements," Bureau of Standards circular C74, U. S. Government Printing Office, Washington, D. C., p. 252; 1937.
[7] F. E. Terman, *op. cit.*, p. 129.
[8] "Reference Data for Radio Engineers," *op. cit.*, p. 131.
[9] F. E. Terman, *op. cit.*, pp. 78–80, 129.

$$R_c = (n\pi d)\phi f^{1/2} d_0^{-1}(1/12) \times 10^{-3}$$
$$= (0.083 \times 10^{-3})(\phi/nd_0)n^2\pi d f^{1/2} \text{ ohms per axial inch,} \quad (15)$$

$$R_s = \frac{9.37 n^2 b^2 (d/2)^4 (1.724 f)^{1/2}}{b\left[D^2(b+d)/8\right]^{4/3}} (\rho_s/\rho_{cu})^{1/2}$$
$$\times 10^{-4} \text{ ohms per axial inch.} \quad (16)$$

The unloaded Q is given by:[10]

$$Q_u = \beta/2\alpha = 2\pi f_0 L/(R_c + R_s), \quad (17)$$

the last form being the value when dielectric losses are neglected. Note that the Q is the same as that approached by a lumped LC resonant circuit as the losses in the capacitor and connections approach zero.

For the resonator with copper coil and copper shield,

$$Q_u = 600 \frac{(d/D) - (d/D)^2}{(\phi/nd_0)(1 + R_s/R_c)} D f_0^{1/2}. \quad (18)$$

Instead of determining the proximity factor ϕ for each case, it is sufficiently accurate to write:

$$\phi/(nd_0) = \phi\tau/d_0 = 3.7, \quad (19)$$

when d_0/τ lies within the range given in the conditions for (1).

From these there can be derived two practical formulas for Q with accuracy of about ± 10 per cent when $0.45 < (d/D) < 0.6$ and $(b/d) > 1.0$.

$$Q_u = 220 \frac{(d/D) - (d/D)^2}{1.5 + (d/D)^2} D f_0^{1/2}, \quad (20)$$

$$Q_u = 50 D f_0^{1/2}. \quad (21)$$

The Q in these formulas has been arbitrarily reduced 10 per cent below the theoretical value to allow for imperfect surface conditions of the conductor. Tabulation of errors of (20) vs (18) (reduced 10 per cent) for the ranges of d/D and b/d listed for (20) shows them to give substantially equal results, and similarly for (21) vs (20).

A useful relationship is that of Q_u and volume of shield. When $0.4 < d/D < 0.6$ and $1.0 < b/d < 3.0$;

$$Q_u = 50\,(\text{vol})^{1/3} f_0^{1/2}. \quad (22)$$

Proportional Relationships

The energy considerations of resonant lines are given in a previous reference.[3] Suppose a series of resonators is considered that has various sizes and resonance frequencies, but in which the geometrical porportions are all identical. Consequently, the design parameters d/D, b/d, and $nd_0 = d_0/\tau$ are the same for all the resonators. Let the input and output loadings be equal or matched, so that there is no mismatch loss.

Then the following proportionalities and equations can be written:

$$Q_u \propto f_0^{1/2} d, \quad (23)$$

$$P_c = 2P_m Q_d/Q_u, \quad (24)$$

$$(\text{temp rise}) \propto P_c/d^2, \quad (25)$$

$$(\text{vol}) \propto d^3. \quad (26)$$

Various combinations can be made of the above expressions:

$$(\text{temp rise}) \propto P_m Q_d (\text{vol})^{-1} f_0^{-1/2}. \quad (27)$$

The voltage gradient at the open-circuited end, and similarly between turns at the short-circuited end, is:

$$(\text{volt grad}) \propto V_{oc}/d = Z_0 I_{sc}/d$$
$$\propto (P_m Q_d)^{1/2}\left[(\text{vol}) f_0\right]^{-1/2} \quad (28)$$

For a given P_m and Q_d there results:

A) temperature rise constant;

$$\left.\begin{array}{l}(\text{vol}) \propto f_0^{-1/2} \\ Q_u \propto f_0^{1/3} \\ (\text{volt grad}) \propto f_0^{-1/4}\end{array}\right\} \quad (29)$$

B) unloaded Q constant;

$$\left.\begin{array}{l}(\text{vol}) \propto f_0^{-3/2} \\ (\text{temp rise}) \propto f_0 \\ (\text{volt grad}) \propto f_0^{1/4}\end{array}\right\}. \quad (30)$$

The proportionalities in (29) and (30) are plotted in Fig. 8. A certain frequency ratio $f_0/f_1 = 1.0$ is shown, above which the temperature rise is indicated as constant, while the relative volume, voltage gradient and Q_u are plotted vs frequency ratio. In this region the unloaded Q is adequate, but temperature rise is the limiting factor. The volume is chosen according to the power rating P_m and the loaded Q.

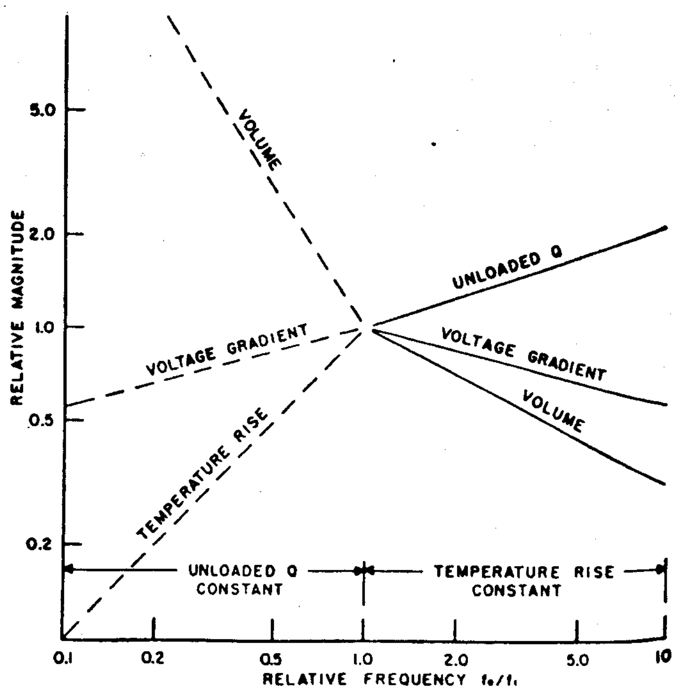

Fig. 8—Relative characteristics of helical resonators.

[10] "Reference Data for Radio Engineers," *op. cit.*, pp. 553, 575.

Below the frequency ratio $f_0/f_1 = 1.0$ the unloaded Q is the limiting factor, because it cannot be allowed to decrease indefinitely. The lower the unloaded Q the greater the dissipation loss. Then the efficiency of the system would become lower than is desirable, or, in other words, the transducer loss would be excessive. In this region, as shown in Fig. 8, the volume increases rapidly.

Power Rating

The power-handling capacity can be estimated by use of (24) and the dissipation from the surface of the shield:

$$P_c = k \text{ (shield area) watts.} \tag{31}$$

When $d/D = 0.55$, $b/d = 1.5$ and $B = b + D/2$ the entire shield area is 5.8 D^2. Then when the top and bottom of the shield are closed and no ventilating holes are provided

$$P_m = 145kD^2 f_0^{1/2}/Q_d \text{ watts.} \tag{32}$$

The value of k depends on the design and materials of the resonator. A conservative value is believed to be $k = 0.4$ watt per square inch.

For a zero- or infinite-impedance generator properly loaded by the cavity and its load:

$$P_a = 290kD^2 f_0^{1/2}/Q_e \text{ watts.} \tag{33}$$

Correction

R. S. Colvin, of the Radio Propagation Laboratory, Stanford University, Stanford, Calif., has brought the following correction to the attention of Peter D. Strum, author of "Considerations in High-Sensitivity Microwave Radiometry," which appeared on pages 43–50 of the January, 1958, issue of Proceedings.

In Table I, page 46, the last line, right-hand column, should read "$1/\sqrt{2}$," not "$1/\sqrt{\pi}$," as shown.