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What quantum computers may tell us about quantum mechanics

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Quantum mechanics occupies a unique position in the history of science. It has survived all experimental tests to date, culminating with the most precise comparison of any measurement to any theory – a 1987 measurement of the electron's magnetic moment, or gyromagnetic ratio $g_e = 2.00231930439$ (Van Dyck *et al.* 1987), agreeing with QED theory to 12 digits. Despite this and other dramatic successes of quantum mechanics, its foundations are often questioned, owing to the glaring difficulties in reconciling quantum physics with the classical laws of physics that govern macroscopic bodies. If quantum mechanics is indeed a complete theory of nature, why does it not apply to everyday life? Even Richard Feynman (1982), a fierce defender of quantum mechanics, memorably stated that:

> We have always had a great deal of difficulty in understanding the world view that quantum mechanics represents . . . Okay, I still get nervous with it . . . It has not yet become obvious to me that there is no real problem. I cannot define the real problem, therefore I suspect there's no real problem, but I'm not sure there's no real problem.

In the dawn of the twenty-first century, John A. Wheeler's big question "Why the quantum?" has returned to the forefront of physics with full steam. Advances in experimental physics are beginning to realize the same thought-experiments that proved helpful to Einstein, Bohr, Heisenberg, Schrödinger, and the other founders of quantum mechanics. The current progression toward nanotechnology, where electronic computing and storage media are being miniaturized to the atomic scale, is beginning to confront quantum-mechanical boundaries, as foreseen in Feynman's early charge, "There's plenty of room at the bottom" (Feynman 1960). While many of these effects are inhibiting the continued miniaturization, new opportunities

such as quantum information processing are arising (Nielsen and Chuang 2000), providing a great incentive to build devices that may not only eclipse the performance of current devices, but also may push quantum theory to its limits. From the standpoint of physics, the new field of quantum information science gives us a very useful language with which to revisit the fundamental aspects of quantum mechanics.

Quantum information processing

Information theory began in the mid twentieth century, with Claude Shannon's seminal discovery of how to quantify classical information (Shannon 1948). Shannon's bit, or binary digit, became the fundamental unit, providing a metric for comparing forms of information and optimizing the amount of resources needed to faithfully convey a given amount of information, even in the presence of noise. Shannon's pioneering work led to the experimental representation of bits in nature, from unwieldy vacuum tubes in the mid twentieth century to the modern VLSI semiconductor transistors of under 0.1 μm in size. Under this impressive progression of technology, we have enjoyed an exponential growth in computing power and information processing speed given by the familiar "Moore's law," where computer chips have doubled in density every year or two.

But this growth will not continue indefinitely. As bits continually shrink in size, they will eventually approach the size of individual molecules – by the year 2020 if the current growth continues. At these nanometer-length scales, the laws of quantum mechanics begin to hold sway. Quantum effects are usually thought of as "dirty" in this context, causing unwanted tunneling of electrons across the transistor gates, large fluctuations in electronic signals, and generally adding noise. However, Paul Benioff and Richard Feynman showed in the early 1980s that quantum-mechanical computing elements such as single atoms could, in principle, behave as adequate electronic components not hampered by dirty quantum effects (Benioff 1980, 1982; Feynman 1982). They even discussed using "quantum logic gates" largely following the laws of quantum mechanics, and Feynman became interested in the idea of using model quantum systems to simulate efficiently other intractable quantum systems (Feynman 1982).

Soon after, David Deutsch went a step further by using the full arsenal of quantum mechanical rules. Deutsch proposed that the phenomenon of quantum superposition be harnessed to yield massively parallel processing – computing with multiple inputs at once in a single device (Deutsch 1985). Instead of miniaturizing chip components further, Deutsch posed an end-run around the impending limits of Moore's law by taking advantage of different physical principles underlying these components.

Whereas Shannon's classical bit can be either 0 or 1, the simplest quantum-mechanical unit of information is the quantum bit or *qubit*, which can store superpositions of 0 and 1. A single qubit is represented by the quantum state

$$\Psi_1 = \alpha|0\rangle + \beta|1\rangle, \tag{17.1}$$

where α and β are the complex amplitudes of the superposition. The states $|0\rangle$ and $|1\rangle$ may represent, for example, horizontal and vertical polarization of a single photon, or two particular energy levels within a single atom. The standard (Copenhagen) rules of quantum mechanics dictate that: (a) the time development of amplitudes α and β is described by the Schrödinger wave equation, and (b) when the above quantum bit is measured, it yields either $|0\rangle$ or $|1\rangle$ with probabilities given by $|\alpha|^2$ and $|\beta|^2$, respectively. The measurement of a quantum bit is much like flipping a coin – the results can only be described within the framework of probabilities.

Hints of the power of quantum computing can be seen by considering a register of many qubits. In general, N qubits can store a superposition of all 2^N binary numbers:

$$\Psi_N = \gamma_0|000\cdots0\rangle + \gamma_1|000\cdots1\rangle + \cdots + \gamma_{2^N-1}|111\cdots1\rangle. \tag{17.2}$$

To appreciate the power of this exponential storage capacity, note that with merely $N = 300$ quantum bits, the most general quantum state requires over 10^{90} amplitudes. This is more than the number of fundamental particles in the universe!

When a quantum computation is performed on a quantum superposition, each piece gets processed in superposition. For example, quantum logic operations can shift all the qubits one position to the left, equivalent to multiplying the input by two. When the input state is in superposition, all inputs are simultaneously doubled in one step (see Fig. 17.1a). After this quantum parallel processing, the state of the qubits must ultimately be measured. Herein lies the difficulty in designing useful quantum computing algorithms: according to the laws of quantum mechanics, this measurement yields just one answer out of 2^N possibilities; worse still, there is no way of knowing which answer will appear. Apparently quantum computers cannot compute one-to-one functions (where each input results in a unique output as in the doubling algorithm above) any more efficiently than classical computers.

The trick behind a useful quantum computer algorithm involves the phenomenon of quantum interference. Since the amplitudes $\gamma_0, \gamma_1, \ldots \gamma_{2^N-1}$ in the superposition of eqn (17.2) evolve according to a wave equation, they can be made to interfere with each other. In the end, the parallel inputs are processed with quantum logic gates so that almost all of the amplitudes cancel, leaving only a very small number of answers, or even a single answer, as depicted in Fig. 17.1b. By measuring this answer (or repeating the computation a few times and recording the distribution of

unsorted databases faster than any search conducted on a classical computer (Grover 1997). The happy result of this ßurry of activity is that scientists, mathematicians, engineers, and computer scientists are now studying and learning quantum physics, and their language is quantum information science.

Useful quantum algorithms such as ShorÕs algorithm are not plentiful, and it is unknown how many classes of problems will ultimately beneÞt from quantum computation. In pursuit of useful quantum algorithms, itÕs natural to investigate what makes a quantum computer powerful. The answer to this question may not only guide us toward new applications of quantum information science, but may also provide alternate views of the quantum physics underlying these devices.

Quantum entanglement

The implicit parallelism in quantum superpositions is not revolutionary by itself. Indeed, there are many classical wavelike phenomena and analog processing models that involve superposition and interference. The new ingredient offered by quantum superpositions such as eqn (17.2) is that it takes 2^N amplitudes to describe the state of only N qubits. The general state of a quantum computer (eqn (17.2)) exhibits a property not found in classical superpositions, quantum entanglement. Entanglement refers to the fact that eqn (17.2) cannot in general be written as a direct product state of the N individual qubits state, which would require only $2N$ amplitudes:

$$\Psi_N^{prod} = (\alpha_1|0\rangle + \beta_1|1\rangle)\ (\alpha_2|0\rangle + \beta_2|1\rangle)\ \cdots\ (\alpha_N|0\rangle + \beta_N|1\rangle). \quad (17.3)$$

The concept of quantum entanglement neatly combines the two properties of quantum mechanics Ð superposition and measurement Ð that are by themselves unremarkable, but taken together cause all the usual interpretive conundrums of quantum mechanics. Schrödinger (1935) himself said, ÒI would not call [entanglement] one but rather the characteristic trait in quantum mechanics, the one that enforces an entire departure from all our classical lines of thought.Ó Yet entanglement seems to be one of the most misunderstood concepts in quantum mechanics. There seem to be many levels of deÞnition, with their own supporting assumptions. Below, several possible deÞnitions of entanglement are considered.

The classic case of quantum entanglement is the thought experiment originally proposed by Einstein, Podolsky, and Rosen (Einstein et al. 1935). EPR posed a quantum state of two particles expressed in position space as

$$\psi(x_1, x_2) = \frac{1}{2\pi} \int e^{j(x_1 - x_2 - s)p/\hbar}\ dp = \delta(x_1 - x_2 - s), \quad (17.4)$$

where $\delta(x)$ is the Dirac-delta function. The particles are always found to be separated in space by s when their positions are measured, yet they are also found to have precisely opposing momenta (seen by Fourier transforming eqn (17.4)). David Bohm's discrete version of the EPR state (Bohm 1951) is the familiar spin-0 particle decaying into two spin-$\frac{1}{2}$ daughter particles (qubits), represented by the spinor quantum state

$$\psi(S_1, S_2) = |\uparrow\rangle_1 |\downarrow\rangle_2 - |\downarrow\rangle_1 |\uparrow\rangle_2, \tag{17.5}$$

where S_1 and S_2 are the spins of the two particles, each taking on one of the two values $\uparrow$ or $\downarrow$. In both cases (eqns (17.4) and (17.5)), the overall quantum state cannot be written as a direct product state of its constituents, and the 'essence' of quantum mechanics in these states is the fact that there their correlation is definite, yet the state of the individual particles is not definite. It's tempting to thereby define entanglement as follows:

Definition 1   An entangled state is a quantum state that is not separable.

(For mixed states, this definition can be extended by requiring inseparability of the density matrix.) But this definition is misleading. While the right-hand side of eqn (17.5) certainly cannot be expressed as a direct product state of the spins, the left-hand side of the equation, describing the same state, is obviously not entangled – it is just the simple lone state $\psi(S_1, S_2)$. For example, in the ground hyperfine states of the hydrogen atom, the entangled singlet state of electron and proton spin is identical to the same state in the usual coupled basis $|J = 0, m_J = 0\rangle$. Many therefore dismiss the whole notion of entanglement as simply a choice of basis. However, entanglement should not only reflect a nonseparable quantum state, but one in which independent quantum measurements on the individual constituents have taken (or will take) place. This measurement naturally selects the uncoupled basis. It might be unsettling to define a quantity that depends on what the experimenter has done (or will do). But this is exactly how most of us interpret quantum mechanics already.

What makes entanglement interesting is that in almost all cases of quantum states expressed following Definition 1, such as hydrogen ground states in the uncoupled basis, it is virtually impossible to measure particular constituents without directly affecting the others. Unfortunately, it would be quite difficult to prepare a hydrogen atom in the singlet state and subsequently measure the electron spin without affecting the proton spin or vice versa. So we might refine the definition in terms of these measurements:

Definition 2   An entangled state is one that is not separable, where measurements are performed on one constituent without affecting the others.

In order to verify the correlations of the subsystems, there must not be much technical noise associated with the measurement. That is, the detection process

itself should not change the quantum state – apart from the usual "wave function collapse" that occurs when a superposition is measured. To be more precise, we require that the probability distribution of measurement results accurately reflect the amplitudes of the original quantum states, and if a subsystem is prepared in a given eigenstate of the measurement operator, our detector should faithfully indicate so. It's reasonable to assume that we cannot tell the difference between a detector that randomly gives incorrect results and a detector that actually influences the quantum state of the system in a random way. Both shortfalls can be lumped into a single parameter known as the detector quantum efficiency, defined as the probability that the detector accurately reflects a measurement of any previously prepared quantum eigenstate.

Definition 3 An entangled state is one that is not separable, where highly quantum-efficient measurements are performed on one constituent without affecting the others.

A quantum computer is nothing more than a device capable of generating an arbitrary entangled state following Definition 3. If the quantum computer consists of N qubits, then the probability that the final measurement accurately reflects the underlying quantum state is η^N, where η is the detector efficiency per qubit. For large numbers of qubits, this requires extremely high detector efficiencies in order to give a reasonable success probability. Even for a 99% efficient detector with each of 1000 qubits, the probability that the complete measurement is not plagued by an error is only 0.00004.

A more strict definition of entanglement would rule out any possibility of inter-action between the constituents during a measurement. This would require that the two subsystems be separated by a spacelike interval (given that we do not abandon relativity). In fact, this condition is the basis for the proof by John Bell that quantum mechanics is an inherently nonlocal theory, and that any extension to quantum mechanics (e.g., involving unobserved "hidden" variables) must itself be nonlocal (Bell 1965). Measurements of Bell's inequality violations are thus very useful measures of entanglement.

Definition 4 An entangled state is one that is not separable, where highly quantum-efficient measurements are performed on one constituent without affecting the others, and where the constituents are spacelike separated during the measurement time.

To date, entangled states following this most strict definition have not yet been created, and no full experimental test of Bell's inequality has been performed (however, see Fry et al. (1995)). Entangled states following Definitions 2 and 3 have been created, with a consequent violation of a Bell's inequality under relaxed conditions ("loopholes"). A series of experiments with optical parametric

downconversion have demonstrated spacelike entanglement with poor detectors (Definition 2) (Weih et al. 1998), and an experiment with two trapped atoms has demonstrated entanglement with efficient detectors but without spacelike separations (Definition 3) (Rowe et al. 2001).

In general, there is no known measure of how much entanglement a given quantum state possesses. An important exception is for the case of pure quantum states that can be represented by a state vector or wave function. Here, the amount of entanglement can be mathematically described as the gain in von Neumann entropy of the state when only a subsystem is considered. This is reasonable, as any pure quantum state has zero entropy, and only when the state is separable does the entropy remain zero when one subsystem is traced over. It is interesting to apply this quantification of entanglement to simple quantum states such as the two entangled states below:

$$A = \frac{+}{\overline{2}} \tag{17.6}$$

$$B = \frac{+\quad+\quad+\quad+\quad+}{\overline{6}} \tag{17.7}$$

Even though state A appears to have a stronger correlation between the four spins, when a trace is performed over any two spins, state B has slightly more entropy than state A, so B is more entangled than A. This definition of entanglement for pure quantum states highlights a peculiar feature of quantum mechanics: the entropy of a quantum subsystem can be more than the entropy of the complete quantum system. This is in stark contrast to classical systems, where entropy of the whole can only be greater than or equal to the sum of the entropies of the individual parts.

Quantum computer hardware

The more strict definitions of entanglement (3 and 4 above) required for a large-scale quantum computer rules out most physical systems. This can be seen by considering the chief hardware requirements for a quantum information processor (DiVincenzo 2000):

 (i) arbitrary unitary operators must be available and controlled to launch an initial state to an arbitrary entangled state (eqn (17.2)), and
(ii) measurements of the qubits must be performed with high quantum efficiency.

From (i), the qubits must be well isolated from the environment to ensure pure initial quantum states and preserve their superposition character, but they must also interact strongly between one another in order to become entangled. On the other hand, (ii)

calls for the strongest possible interaction with the environment to be switched on at will. The most attractive physical candidates for quantum information processors are thus fairly exotic physical systems offering a high degree of quantum control.

A collection of laser-cooled and trapped atomic ions represents one of the few developed techniques to store qubits and prepare entangled states of many qubits (Cirac and Zoller 1995; Monroe et al. 1995; Wineland et al. 1998). Here, electromagnetic Þelds conÞne individual atoms in free space in a vacuum chamber, and when multiple ions are conÞned and laser-cooled, they form simple stationary crystal structures given by the balance of the external conÞning force of the trap with the mutual repulsion of the atoms (Fig. 17.2). Qubits are effectively stored in internal electronic states of the atoms, typically the same long-lived hyperÞne states that are used in atomic clocks. When appropriate laser radiation is directed to the atomic ions, qubit states can be coherently mapped onto the quantum state of collective motion of the atoms and subsequently mapped to other atoms. A single normal mode of collective crystal motion thus behaves as a Òquantum data-bus,Ó allowing quantum information to be shared and entangled between remote atomic qubits in the crystal. Finally, the internal states of individual trapped ions can be measured with nearly 100% quantum efÞciency (Blatt and Zoller 1988) by applying appropriate laser radiation and collecting ßuorescence, as in Fig. 17.2. In certain species atoms, a ÒcyclingÓ transition allows a large amount of ßuorescence to result from one qubit state, while the other remains dark.

Quantum logic gates have been demonstrated with up to four trapped atomic ion qubits, resulting in the generation of particular four-qubit entangled states such as eqn (17.6) (Sackett et al. 2000). While this scheme is scalable to arbitrarily large numbers of qubits in principle, the main problems deal with control of the collective motion of the atoms. As more qubits are added to the collection, the density of motional states balloons, and isolation of a single mode of motion (e.g., the center-of-mass) becomes even more slow and difÞcult (Wineland 1998). Moreover, external noisy electric Þelds tend to compromise the motional coherence of large numbers of trapped atomic ions (Turchette et al. 2000a). A promising approach that attacks both problems is the quantum CCD, where individual atomic ions are entangled as above, but only among a small collection (under 10) of atomic ions in an ÒaccumulatorÓ (Kielpinski et al. 2002). To scale to larger numbers, individual atoms are physically shuttled between the accumulator and a ÒmemoryÓ reservoir of trapped atom qubits. This can be done quickly with externally applied electric Þelds in elaborate ion trap electrode geometries. The central features of the quantum CCD are that trapped ion shuttling can be done without perturbing the internal qubits, and the motional quantum state of the ions factors from the internal qubit states following quantum gate operation. In order to quench this extra motional energy for subsequent logic gates, ancillary ions in the accumulator can be

Figure 17.2. Spatial image of two trapped cadmium atomic ions from the University of Michigan Ion Trap Group (Blinov et al. 2002). Resonant laser radiation near 215 nm illuminates the atoms, and an imager collects the ultraviolet fluorescence. This image was integrated for about 1 s. The atoms are separated by approximately 2 μm, a balance between the external confining force and the Coulomb repulsion. The breadth of each atom is consistent with diffraction from the imaging optics, and the Airy rings are visible around each atom. The confinement electrodes (not shown at this scale) have a characteristic dimension of 200 μm.

laser-cooled in between gate operations; thus the qubit ions are sympathetically cooled through their strong Coulomb interaction with these extra refrigerator ions (Larson et al. 1986; Blinov et al. 2002).

Other potential quantum information processor candidates (Monroe 2002) include trapped atoms in optical lattices, trapped photons (cavity QED), and nuclear magnetic resonance (NMR) techniques applied to low temperature samples Ð nearly identical to the ion trap concept described above. Much less is clear in the domain of many solid-state systems, where quantum mechanics plays only a small role. However, there is exciting current research in exotic condensed-matter systems such as semiconductor quantum dots (Stievater et al. 2001) and superconducting current loops (van der Wal et al. 2000; Friedman et al. 2002) and charge pumps (Nakamura 1999; Vion 2002), which may some day allow the scale-up to a large-scale quantum information processor.

Outlook

There is a proliferation of quantum mechanical interpretations all attempting to address the conceptual problems unifying quantum mechanics with quantum measurement Ð the so-called measurement problem that plagues the conventional Copenhagen interpretation used by the vast majority of physicists. While current experiments are very far from demonstrating useful quantum information processing, some systems may ultimately put us in a position of questioning (or more likely ruling out) these alternatives to quantum mechanics.

The most popular alternative quantum views include Bohmian mechanics Ð a nonlocal hidden-variables theory that at least removes indeterminism from quantum mechanics (Albert 1994); the many-worlds interpretation proposing that quantum measurements cause the universe to bifurcate (Everett 1957); the consistent or decoherent histories approach (Grifþths 2001), and the transactional interpretation of quantum mechanics (Cramer 1988). Perhaps the most popular melding of quantum mechanics and quantum measurement is the theory of decoherence (Zurek 1982, 1991). Decoherence theory applies the usual quantum mechanics to a closed system, but when the uncountable degrees of freedom of the environment are inevitably coupled into that system, via noise or a measurement, entanglements form between the system and environment. Now when we perform a trace over the environmental degrees of freedom, we þnd that the coherence in quantum mechanics decays, or pure states of a closed quantum system continuously evolve into mixtures. Decoherence formalism is a useful method of calculating the dissipation expected in quantum systems when environmental couplings are known, but it certainly does not address the quantum measurement problem. This would be akin to claiming that NewtonÕs law of gravitational attraction $F = Gm_1 m_2 / r^2$ explains the

origin of gravity. In fact, nearly all of the alternative interpretations of quantum mechanics predict the same answers for any conceivable experiment. While some versions have a satisfactory feel to them Ð perhaps by removing the observer from the theory (Goldstein 1998) Ð these differing frameworks might seem unremarkable to the experimentalist.

There is at least one alternative to quantum mechanics that is testable. It posits that quantum mechanics and classical mechanics are just two limits of the same underlying theory. Small systems such as isolated atoms and electrons are well approximated by quantum mechanics, while large systems like cats are well approximated by classical mechanics. Such a theory predicts a frontier between these two limits where new physics may arise. One example is a class of Òspontaneous wave function collapseÓ theories; the most popular having been put forth by Ghirardi, Rimini, and Weber (GRW) in the last decades (Ghirardi et al. 1986; Bell 1987; Pearle 1993). The GRW theory attempts to meld quantum and classical mechanics by adding a nonlinear stochastic driving field to quantum mechanics that randomly localizes or collapses wave functions. This localization acts with an effective spatial dimension a, and the frequency of the collapses is proportional to a rate times the number of degrees of freedom N in the system. The fundamental constants a and are chosen such that the average time of collapse of simple systems like a single atom or electron is very long, while the average time of collapse of a macroscopic superposition of a body with 10^{20} degrees of freedom is unobservably short (favored values of a and are approximately 10^{-5} cm and 10^{-16} Hz, respectively). Admittedly, such a phenomenological theory is not very plausible, but the ad hoc details of GRWÕs proposal are not the main point. What makes their theory remarkable is that it is testable. Stochastic collapses predicted by GRW indeed imply an upper limit on the size of a quantum computer.

Experiments that may test spontaneous wave function collapse theories are naturally the same systems that are considered as viable future quantum computers. A review on the state-of-the-art in Òlarge superpositionsÓ is considered by Leggett (2002), including an exhaustive definition of what constitutes a Òdegree of freedomÓ so critical to the GRW theory. The more notable systems include quantum optics systems of trapped ions (Monroe et al. 1996; Myatt et al. 2000; Turchette et al. 2000b) and the related system of cavity QED (Brune 1996); and superconducting systems of quantum dots (Nakamura et al. 1999; Vion et al. 2002) and SQUIDs (van der Waal et al. 2000; Friedman 2002). The quantum optics systems are complementary to the condensed matter systems in the context of attacking the GRW wave function collapse frontier. The superconducting systems deal with superpositions of supercurrents or numbers of Cooper-paired electrons, boasting a very large number N of degrees of freedom. However, access to these individual degrees of freedom through highly efficient measurements has not been demonstrated. This masks the

underlying entanglement in the system (see Deﬁnition 3 above), admitting a more classical-like description of the observed phenomena. Quantum optics systems, on the other hand, offer highly efﬁcient measurements, but only with a small value for N. All the above systems are prime candidates for quantum computing hardware, and as more qubits are entangled in these (or any quantum hardware), so too will the frontiers of GRW collapse be pushed back.

Of the three possible results in the quest to build a quantum computer, two are tantalizing: either a fully blown large-scale quantum computer will be built, or the theory of quantum mechanics will be found to be incomplete. The third possibility, that the technology will never reach the complexity level required for either of the ﬁrst possibilities due to economic constraints, has nothing to do with physics, but is probably favored by the majority of physicists. Indeed, itÕs amusing to see physicists bristle when confronted with the notion of a macroscopic quantum state Ð aÒSchr¬dinger cat.Ó In A Brief History of Time, Stephen Hawking quips that ÒWhenever I hear a mention of that cat, I reach for my gun.Ó Even Schr¬dinger himself labeled his famous cat as ridiculous, and was so disturbed at this logical path of quantum mechanics, that he switched ﬁelds altogether. It is this steadfast parochial view that suggests that we should continue to probe foundational aspects of quantum mechanics, even if the result is only a full-scale quantum information processor.

Acknowledgments

References

Albert, DZ (1994) BohmÕs alternative to quantum mechanics. Scient. Am. 270, 32.
Bell, JS (1965) On the EinsteinÐPodolskyÐRosen paradox. Physics 1, 195.
    (1987) Speakable and Unspeakable in Quantum Mechanics. Cambridge: Cambridge University Press.
Benioff, P (1980) The computer as a physical system: a microscopic quantum mechanical model of computers as represented by Turing machines. J. Stat. Phys. 22, 563.
    (1982) Quantum mechanical Hamiltonian models of Turing machines that dissipate no energy. Phys. Rev. Lett. 48, 1581.
Blatt, R and Zoller, P (1988) Quantum jumps in atomic systems. Eur. J. Phys. 9, 250.
Blinov, B, et al. (2000) Sympathetic cooling of trapped Cd^+ isotopes. Phys. Rev. A 65, 040304.
Bohm, D (1951) Quantum Theory. New York: Dover.
Brune, M, et al. (1996) Observing the progressive decoherence of the ÔmeterÕ in a quantum measurement. Phys. Rev. Lett. 77, 4887.
Cirac, I and Zoller, P (1995) Quantum computations with cold trapped ions. Phys. Rev. Lett. 74, 4091.

Cramer, JG (1988) An overview of the transactional interpretation of quantum mechanics. *Int. J. Theor. Phys.* 27, 227.

Deutsch, D (1985) Quantum theory, the Church–Turing principle and the universal quantum computer. *Proc. Roy. Soc. Lond.* A400, 97.

DiVincenzo, D (2000) The physical implementation of quantum computation. *Fortschr. Phys.* 48, 771.

Einstein, A, *et al.* (1935) Can quantum-mechanical description of reality be considered complete? *Phys. Rev.* 47, 777.

Everett, H (1957) Relative state formulation of quantum mechanics. *Rev. Mod. Phys.* 29, 454.

—— (1960) http://www.zyvex.com/nanotech/feynman.html.

Feynman, RP (1982) Simulating physics with computers. *Int. J. Theor. Phys.* 21, 467.

Friedman, JR, *et al.* (2002) *Nature* 406, 43.

Fry, ES, *et al.* (1995) Proposal for loophole-free test of the Bell inequalities. *Phys. Rev. Lett.* 52, 4381.

Ghirardi, GC, *et al.* (1986) Unified dynamics for microscopic and macroscopic systems. *Phys. Rev.* D34, 470.

Goldstein, S (1998) Quantum theory without observers. *Phys. Today* March, 42; April, 38.

Griffiths, RB (2001) *Consistent Quantum Theory.* Cambridge: Cambridge University Press.

Grover, L (1997) Quantum mechanics helps in searching for a needle in a haystack. *Phys. Rev. Lett.* 79, 325.

Kielpinski, D, et al. (2002) Architecture for a large scale ion-trap quantum computer. *Nature* 417, 709.

Larson, DJ, *et al.* (1986) Sympathetic cooling of trapped ions: a laser-cooled two-species nonneutral ion plasma. *Phys. Rev. Lett.* 57, 70.

Leggett, AJ (2002) Testing the limits of quantum mechanics: motivation, state of play, prospects. *J. Phys. Condens. Matter* 14, R415.

Monroe, C (2002) Quantum information processing with atoms and photons. *Nature* 416, 238.

Monroe, C, *et al.* (1995) Demonstration of a universal quantum logic gate. *Phys. Rev. Lett.* 75, 4714.

—— (1996) A Schrödinger cat superposition state of an atom. *Science* 272, 1131.

Myatt, C, *et al.* (2000) Decoherence of quantum superpositions coupled to engineered reservoirs. *Nature* 403, 269.

Nakamura, Y, *et al.* (1999) Coherent control of macroscopic quantum states in a single Cooper-pair box. *Nature* 398, 786.

Nielsen, MA and Chuang, IL (2000) *Quantum Computation and Quantum Information.* Cambridge: Cambridge University Press.

Pearle, P (1993) Ways to describe dynamical state-vector reduction. *Phys. Rev.* A48, 913.

Rivest, R, *et al.* (1978) A method for obtaining digital signatures and public-key cryptosystems. *Comm. Ass. Comptg Machinery* 21, 120.

Rowe, MA, *et al.* (2001) Experimental violation of a Bell's inequality with efficient detectors. *Nature* 409, 791.

Sackett, C, *et al.* (2000) Experimental entanglement of four particles. *Nature* 404, 256.

Schrödinger, E (1935) Discussion of probability relations between separated systems. *Proc. Camb. Philos. Soc.* 31, 555.

Shannon, CE (1948) A mathematical theory of communication. *Bell System Tech. J.* 27, 379; 623.

Shor, P (1997) Polynomial-time algorithms for prime factorization and discrete logarithms on a quantum computer. *SIAM J. Comp.* 26, 1484.

Stievater, T, et al. (2001) Rabi oscillations of excitons in single quantum dots. *Phys. Rev. Lett.* 87, 133603.

Turchette, Q, et al. (2000a) Quantum heating of trapped ions. *Phys. Rev.* A61, 063418.

(2000b) Decoherence of quantum superpositions coupled to engineered reservoirs. *Phys. Rev.* A62, 053807.

van der Waal, CH, et al. (2000) Quantum superposition of macroscopic persistent-current states. *Science* 290, 773.

Van Dyck, RS, et al. (1987) New high-precision comparison of electron and positron *g* factors. *Phys. Rev. Lett.* 59, 26.

Vion, D, et al. (2002) Manipulating the quantum state of an electrical circuit. *Science* 296, 886.

Weihs, G, et al. (1998) Violation of Bell's inequality under strict Einstein locality conditions. *Phys. Rev. Lett.* 81, 5039.

Wineland, D, et al. (1998) Experimental issues in coherent quantum manipulation of trapped atomic ions. *Nat. Inst. Standards and Tech. J. Res.* 103, 259.

Zurek, WH (1982) Environment-induced superselection rules. *Phys. Rev.* D26, 1862.

(1991) Decoherence and the transition from quantum to classical. *Phys. Today* 44(10), 36.