Discussion: Scaling of decoherence and errors in quantum simulation

We consider the use of a linear crystal of trapped atomic ion spins for the quantum simulation of the Ising spin model with transverse magnetic field

$$H = \sum_{i<j} J_{i,j}\sigma_x^{(i)}\sigma_x^{(j)} + B_y \sum_i \sigma_y^{(i)}, \tag{1}$$

where $\sigma_\alpha^{(i)}$ are the Pauli operators of the i-th spin. Here we derive scaling laws for decoherence and errors in the observation of magnetic phase transitions of the many-body ground state as the number of ions N grows. We consider errors due to nonadiabatic transitions to excited states, the creation of phonons in the system, and the effect of spontaneous Raman scattering from the lasers. As discussed in the main text, the Ising couplings are determined by the collective normal modes of trapped ion motion that are coupled to the spins through optical dipole forces. We consider the transverse motional modes of the linear crystal because they are tightly confined and their mode spacing and bandwidth can be experimentally adjusted. For concreteness, we assume that the lasers couple to only one of the two directions of transverse motion and that they predominantly drive the center-of-mass motional mode at frequency ν, where the corresponding Ising model has a uniform constant coupling rates over all the ion pairs. Similar scaling laws can be derived for other situations.

Laser beams drive stimulated Raman transitions between ground state hyperfine levels with frequency splitting ν_{HF} that form the effective spin-$1/2$ system in each ion. The Raman lasers are detuned from a dipole-allowed electronic transition (radiative linewidth γ and resonant saturation intensity I_{sat}) by an amount Δ, which is assumed to be smaller than the atomic fine structure splitting. The lasers have microwave beatnotes at frequencies $\nu_{HF} \pm \mu$, where μ is set to be near one or more normal mode motional frequencies[1–3]. The excitation of phonons can be avoided by detuning the laser beatnote from the normal mode sideband by an amount large compared to the sideband Rabi frequency, or $\delta \equiv |\mu - \nu| \gg \eta\Omega$. Here, $\eta = \eta_0/\sqrt{N}$ is the rms Lamb-Dicke parameter of the normal mode over all ions, $\eta_0 = \sqrt{\hbar k^2/(4\pi M\nu)}$ is the single ion Lamb-Dicke parameter with M the ion mass and k the wavevector difference of the Raman laser beams along the transverse motion, and $\Omega = (\gamma^2/4\Delta)(I_1/I_{sat})$ is the Raman carrier Rabi frequency in terms of the intensity per ion I_1.

For this predominantly single-mode situation, we are interested in scaling laws for observation of the phase transition from polarization along the effective magnetic field B_y to order according to the Ising couplings $J_{i,j}$, which occurs at the critical point where $B_y^{(crit)} \sim NJ_{rms}$, with $J_{rms} = \sqrt{\sum_{i,j} J_{i,j}^2/N}$ the characteristic Ising coupling[4]. For a fixed frequency bandwidth of all N transverse modes, the typical spacing of adjacent transverse modes scales roughly as $1/N$, so as the other modes encroach upon the mode of interest, the detuning must also be decreased as $\delta = \delta_0/N$, with δ_0 fixed. The probability of phonon excitation p_{ph} during the simulation is given by the squared ratio of the sideband Rabi frequency (spectral width of the sideband transition) to the detuning of the lasers from the sideband:

$$p_{ph} = \left(\frac{\eta\Omega}{\delta}\right)^2 = N\left(\frac{\eta_0\Omega}{\delta_0}\right)^2. \tag{2}$$

We find that in order to keep phonon errors at a fixed level, the carrier Rabi frequency must therefore be slowed down with increasing ion number as $\Omega \sim 1/\sqrt{N}$.

The characteristic Ising coupling J_{rms} and the effective magnetic field B_y (when generated from independent Raman beams with $\mu = 0$ and intensity I_1' per ion) are given in terms of the above parameters

$$J_{rms} = \frac{(\eta\Omega)^2}{2\delta} = \frac{\delta_0 p_{ph}}{2N} = \left(\frac{\eta_0^2\gamma^4}{32 I_{sat}^2 \delta_0}\right)\frac{I_1^2}{\Delta^2} \quad (3)$$

$$B_y = \Omega' = \left(\frac{\gamma^2}{4I_{sat}}\right)\frac{I_1'}{\Delta}. \quad (4)$$

Now we consider the error due to spontantous scattering from the off-resonant Raman beams during the adiabatic control of the Hamiltonian. We assume that the Ising coupling is kept on for a time τ scaled by the inverse of the gap between the (degenerate) ground states and the next excited state(s), and the magnetic field B_y is adiabatically ramped down to track the ground state over this time. The energy gap between the ground and excited state vanishes at the critical field $B_y^{(crit)} = NJ_{rms}$ for a thermodynamic system, but for a finite system of N spins, the gap has a minimum value approaching $B_y^{(crit)}/N^{1/3} = J_{rms}N^{2/3}$ for the long-range Ising model[5,6]. So a conservative estimate of the simulation time can be written

$$\tau = \frac{\beta}{J_{rms}N^{2/3}} = \left(\frac{2\beta}{\delta_0 p_{ph}}\right) N^{1/3} \quad (5)$$

where the constant adiabaticity parameter β is of order unity and indicates the level of error due to diabatic evolution. The off-resonant spontaneous scattering errors from the Ising coupling beams (ϵ_J) and the effective magnetic field Raman beams (ϵ_B)over time τ can now be written,

$$\epsilon_J = \frac{\gamma\tau\Omega}{\Delta} = \left(\frac{8\beta\delta_0 I_{sat}}{\eta_0^2\gamma}\right)\frac{1}{I_1 N^{2/3}} \tag{6}$$

$$\epsilon_B = \frac{\gamma\tau\Omega'}{\Delta} = (\beta\gamma)\frac{N^{1/3}}{\Delta} \tag{7}$$

where in the last expression we have used $B_y = NJ_{rms}$.

Finally, observations on the scaling of these errors with N can be made:

- For fixed total laser power, the beam must be spread out in one dimension to transversely illuminate the ions, implying that $I_1 \sim 1/N$. In this case, for constant phonon error probability p_{ph}, the optical detuning scales as $\Delta \sim 1/\sqrt{N}$ and the resulting spontaneous emission error from the Ising coupling grows slowly as $\epsilon_J \sim N^{1/3}$. On the other hand, the spontaneous emission error from the effective B_y magnetic field generated from Raman beams grows almost linearly as $\epsilon_B \sim N^{5/6}$. However, this error can be eliminated by using a microwave source to provide the effective B_y field, where there is negligible spontaneous emission.

- The spontaneous emission error from the Ising coupling can be made independent of N so long as the total laser power increases as $N^{1/3}$. In this case, the detuning must be slowly decreased as $N^{-1/6}$ to keep the phonon error fixed.

- For fixed detuning Δ, we must have the intensity per ion dropping off as $I_1 \sim 1/\sqrt{N}$ to keep the phonon error p_{ph} fixed, and the total laser power must therefore increase with $\sqrt{N}$. In this case, the spontaneous emission error from the J coupling actually decreases as $\epsilon_J \sim N^{-1/6}$, while the spontaneous emission error from the Raman carrier effective B field scales as $\epsilon_B \sim N^{1/3}$.

- For fixed levels of error due to phonon coupling and diabatic transitions out of the ground state, the required ramping time grows as $\tau \sim N^{1/3}$. Despite this weak scaling with N, other slowly accumulating errors such as ion heating and drifts of (real) magnetic fields or laser intensity could eventually limit the accuracy of quantum simulations in this system.

References

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