Experimental Demonstration of Entanglement-Enhanced Rotation Angle Estimation Using Trapped Ions

V. Meyer,[1] M. A. Rowe,[1] D. Kielpinski,[1] C. A. Sackett,[2] W. M. Itano,[1] C. Monroe,[3] and D. J. Wineland[1]
[1]*Time and Frequency Division, National Institute of Standards and Technology, Boulder, Colorado 80305-3328*
[2]*Physics Department, University of Virginia, Charlottesville, Virginia 22904*
[3]*Department of Physics, University of Michigan, Ann Arbor, Michigan 48109*
(Received 26 February 2001)

We experimentally investigate three methods, utilizing different atomic observables and entangled states, to increase the sensitivity of rotation angle measurements beyond the "standard quantum limit" for nonentangled states. All methods use a form of quantum mechanical "squeezing." In a system of two entangled trapped $^9Be^+$ ions we observe a reduction in uncertainty of rotation angle below the standard quantum limit for all three methods including all sources of noise. As an application, we demonstrate an increase in precision of frequency measurement in a Ramsey spectroscopy experiment.

DOI: 10.1103/PhysRevLett.86.5870 PACS numbers: 42.50.Dv, 03.65.Ud, 32.80.Pj, 39.30.+w

Entanglement has played an important role in elucidating fundamental, and sometimes apparently mysterious, aspects of quantum mechanics [1]. It is an integral part of quantum information processing [2], where potential applications include efficient algorithms for problems that are computationally hard on classical computers. Entanglement can also provide increased sensitivity in quantum-limited measurements; here we report experimental measurements of rotation angle in an atomic ensemble where the observed uncertainty is smaller than can possibly be obtained without entanglement.

Generally, we assume a quantum system where an observable $\tilde{O}$ depends on a system parameter ζ. Using measurements of $\tilde{O}(\zeta)$ to determine ζ, the uncertainty in our determination of ζ for a single measurement is given by

$$\delta\zeta = \frac{\Delta\tilde{O}}{|\partial\langle\tilde{O}\rangle/\partial\zeta|}, \qquad (1)$$

where $(\Delta\tilde{O})^2 \equiv \langle\tilde{O}^2\rangle - \langle\tilde{O}\rangle^2$ is a measure of the rms fluctuations in repeated measurements of $\tilde{O}$. The specific problem we investigate is efficient estimation of spin rotation angle. We consider a system of N spin-1/2 particles with total angular momentum $\boldsymbol{J} = \sum_{i=1}^{N} \boldsymbol{S}_i$, where $\boldsymbol{S}_i$ is the spin of the ith particle. For each spin, $|\downarrow\rangle_i$ and $|\uparrow\rangle_i$ are spin eigenstates with respect to a chosen axis. Rotations of the entire system are characterized by the operator $R = \exp(-i\zeta \boldsymbol{J}\cdot\hat{\boldsymbol{u}})$ for ζ an angle and $\hat{\boldsymbol{u}}$ the axis of rotation. To make the best estimate of ζ, we want to prepare an input state and choose an observable $\tilde{O}$ that will minimize $\delta\zeta$. This problem is analogous to measurements of path-dependent phase differences in a Mach-Zehnder interferometer [3–5] or transition frequencies in spectroscopy [6–8].

For nonentangled spin-1/2 particles, the states which minimize $\delta\zeta$ are (angular momentum) coherent states [9]. Coherent states can be obtained from the state $|\Psi_0\rangle = |\downarrow\rangle_1|\downarrow\rangle_2\cdots|\downarrow\rangle_N = |J = N/2, m_J = -N/2\rangle$ by an overall rotation. In this case $\tilde{O} = \tilde{J}_\perp$ minimizes $\delta\zeta$, where $\tilde{J}_\perp$ is the angular momentum operator perpendicular to $\langle\boldsymbol{J}\rangle$ in the plane of rotation (Fig. 1). Then, $\delta\zeta = \Delta J_\perp/|\langle\boldsymbol{J}\rangle|$ [10]. Assuming no additional sources of error, for coherent states we obtain $\delta\zeta = \delta\zeta_c = 1/\sqrt{N}$, the standard quantum limit (SQL) [3–8].

We examine three proposed methods to reduce $\delta\zeta$ using entangled states. The first uses states well described by "spin squeezing" [3,4,6], as depicted in Fig. 1b. Here, we take $\tilde{O} = \tilde{J}_\perp$ and $\delta\zeta = \Delta J_\perp/|\langle\boldsymbol{J}\rangle|$ as for coherent states. A second method [11–13] has been discussed in the context of a Mach-Zehnder interferometer for bosons, where the angle ζ to be measured is the interferometer phase offset due to unequal arm length. In the spin context here, a state of the form $|\Psi\rangle = |J = N/2, m_J = 0\rangle$ is rotated and subsequently measured with the variance operator $\tilde{V} = \tilde{J}_z^2 - \langle\tilde{J}_z\rangle^2$. A third method [8] uses states of the form $|\Psi\rangle = [|J, -J\rangle + e^{i\beta}|J, +J\rangle]/\sqrt{2} = [|\downarrow\rangle_1|\downarrow\rangle_2\cdots|\downarrow\rangle_N + e^{i\beta}|\uparrow\rangle_1|\uparrow\rangle_2\cdots|\uparrow\rangle_N]/\sqrt{2}$ that are rotated and subsequently measured with the parity operator $\tilde{O} = \tilde{P} = \prod_{i=1}^{N}\sigma_{zi}$, where $\sigma_{zi} = 2S_{zi}/\hbar$ is the Pauli spin operator in the z direction for the ith particle. In experiments

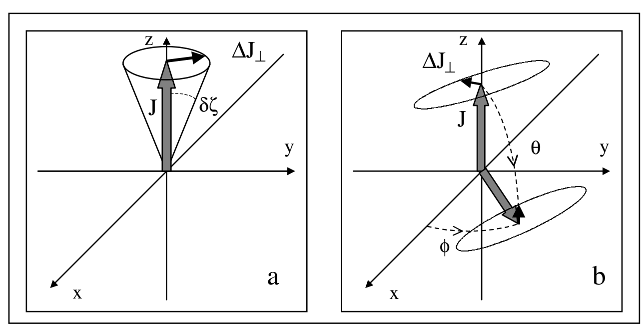

FIG. 1. (a) For spins in a coherent state the uncertainty distribution of the perpendicular spin components $\Delta J_\perp$ is uniform and forms a circle of radius $\sqrt{J/2}$, whereas for entangled spins (b) the distribution of $\Delta J_\perp$ can be "squeezed" in one direction, forming an ellipse. To measure $\Delta J_\perp$, we rotate the spin $\boldsymbol{J}$ into the x-y plane and measure ΔJ_z, observing its oscillation with respect to the phase angle ϕ. From this and the measured value of $|\langle\boldsymbol{J}\rangle|$ we determine $\delta\theta$ (Fig. 2).

 0031-9007/01/86(26)/5870(4)$15.00

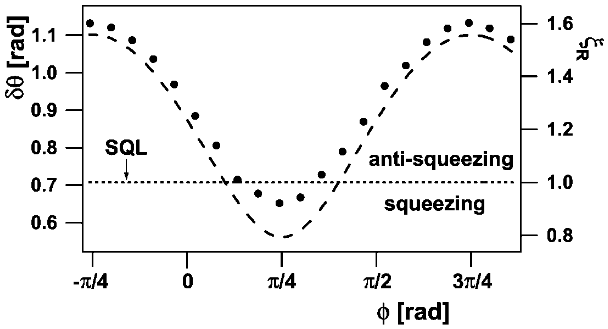

FIG. 2. The ideal phase sensitivity of the state $|\Psi\rangle = \cos(\pi/10)|\downarrow\downarrow\rangle + i\sin(\pi/10)|\uparrow\uparrow\rangle$ is shown as the dashed curve [Eq. (4)]. The full circles are the values determined from 10 000 experiments ($\simeq$1 ms per experiment) per data point. Error bars are smaller than the marker size. The dotted line marks the standard quantum limit $\delta\theta_c = 1/\sqrt{2}$, which is independent of ϕ [7].

with two ions, we use each of these methods and measure a value of $\delta\zeta < \delta\zeta_c$. We quantify these results with the parameter $\xi_R = \delta\zeta/\delta\zeta_c$ [6], related to the entanglement of the system [14]. The minimum possible value of ξ_R is $1/\sqrt{N}$ ($\delta\zeta = 1/N$), the Heisenberg limit [3–8].

The experiments use two $^9Be^+$ ions that are confined along the axis of a miniature linear radio frequency trap [15]. The spectrally resolved $|F = 1, m_F = -1\rangle \equiv |\uparrow\rangle$ and $|F = 2, m_F = -2\rangle \equiv |\downarrow\rangle$ $^2S_{1/2}$ ground-state hyperfine levels of $^9Be^+$ form the basis of an effective spin-1/2 system. Coherent superpositions between $|\downarrow\rangle$ and $|\uparrow\rangle$ are generated by laser-driven two-photon stimulated-Raman transitions [16]. Defining the quantization axis to be the z axis, these operations are equivalent to the spin rotation R for $\hat{\boldsymbol{u}}$ in the x-y plane:

$$|\downarrow\rangle \rightarrow \cos\tfrac{\theta}{2}|\downarrow\rangle - \sin\tfrac{\theta}{2}e^{-i\phi}|\uparrow\rangle, \qquad (2a)$$

$$|\uparrow\rangle \rightarrow \cos\tfrac{\theta}{2}|\uparrow\rangle + \sin\tfrac{\theta}{2}e^{i\phi}|\downarrow\rangle, \qquad (2b)$$

where ϕ is the laser phase and θ is proportional to the duration the laser pulses are applied (Fig. 1b). Using stimulated-Raman transitions that couple to the ions' motion [17,18], we can also realize the entangling operation:

$$|\downarrow\downarrow\rangle \rightarrow |\Psi\rangle \equiv \cos\alpha|\downarrow\downarrow\rangle + i\sin\alpha|\uparrow\uparrow\rangle, \qquad (3)$$

where α is proportional to the laser pulses' duration. At the end of each experiment, we detect the number of ions in the $|\downarrow\rangle$ or $|\uparrow\rangle$ state with state-sensitive fluorescence [19].

For spin squeezing, we take $|\Psi\rangle$ with values of $\alpha \neq M\pi/4$ (M odd) in which case $\langle \boldsymbol{J}\rangle = \langle J_z\rangle = -\cos(2\alpha)$ does not vanish. In the experiment, we extract $\langle \boldsymbol{J}\rangle$ by applying the rotation of Eqs. (2), varying θ, and recording $\langle J_z\rangle$ ("Rabi flopping"). In general $\Delta J_\perp$ depends on ϕ, as indicated in Fig. 1b. To determine $\Delta J_\perp(\phi)$, we rotate $|\Psi\rangle$ into the x-y plane by driving a "$\pi/2$-pulse" on both ions [Eqs. (2) with $\theta = \pi/2$] and measure ΔJ_z for different values of ϕ. This operation preserves the expectation values of $\langle \boldsymbol{J}\rangle$ and $\Delta J_\perp$ in the rotated frame. Therefore we can regard the experiment as measuring the precision of our $\zeta = \theta = \pi/2$ rotation of the spins for the initial state $|\Psi\rangle$; this precision will be optimized for certain values of ϕ. For the ideal case, we have

$$\delta\theta(\phi) = \frac{\Delta J_\perp(\phi)}{|\langle \boldsymbol{J}\rangle|} = \frac{\sqrt{\frac{1}{2}[1 - \sin(2\alpha)\sin(2\phi)]}}{|\cos(2\alpha)|}. \qquad (4)$$

We performed experiments for several values of α, obtaining the highest sensitivity for $\alpha = \pi/10$ (Fig. 2). From the Rabi flopping curves we measure $|\langle \boldsymbol{J}\rangle|$ to be 0.768(2) in this case (ideally, we expect 0.809). The highest measured sensitivities achieved are $\delta\theta_{\min} = 0.65(1)$, or $\xi_R = 0.92(1)$. [Ideally $\delta\theta_{\min} = 0.561$ ($\xi_R = 0.794$)]. An increase in sensitivity over the SQL for two nonentangled spins ($\delta\theta_c = 1/\sqrt{2}$) is visible for a range of about $\pi/4$ rad. The discrepancy with the theoretically expected minimal values is caused primarily by imperfect entangled-state preparation. Note that ideally, for two spins, $\delta\theta_{\min} \rightarrow 1/2$ (the Heisenberg limit) as $\alpha \rightarrow \pi/4$. However, then $\langle \boldsymbol{J}\rangle \rightarrow 0$ so that any added noise prevents achieving this limit.

The improvements in phase sensitivity over the SQL arise from spin squeezing. For angular momentum states, the uncertainty relation $(\Delta J_i)^2(\Delta J_j)^2 \geq \frac{1}{4}\hbar^2|\langle J_k\rangle|^2$ allows for one of the variances $(\Delta J_i)^2$ to be reduced (squeezing) at the cost of $(\Delta J_j)^2$ increasing (antisqueezing). $|\langle J_k\rangle|$ must also shrink. The Hamiltonian $H = \chi J_x^2$ which carries the state $|\downarrow\downarrow\rangle$ into $|\Psi\rangle$ [Eq. (3)] [4,17,20] establishes a correlation between the spins of both particles that results in spin squeezing [4]. The redistributed variances of the spins are indicated as an ellipse in Fig. 1b and allow us to obtain values of $\xi_R < 1$ when ΔJ_i shrinks more than $|\langle J_k\rangle|$. Recently, observations of spin squeezing have been reported in Refs. [21] and [22], but the results were not cast in terms of measured $\delta\zeta$ or ξ_R; therefore, a direct comparison is precluded.

For the maximally entangled state $|\Psi_M\rangle = (|\downarrow\downarrow\rangle + i|\uparrow\uparrow\rangle)/\sqrt{2}$ [$\alpha = \pi/4$ in Eq. (3)], or the state $|J = 1, m_J = 0\rangle$, $\langle \boldsymbol{J}\rangle = 0$ [23] so that the above method is experimentally inaccessible. In these cases, the parity operator $\tilde{P}$ or the variance $\tilde{V}$ can be used to increase the sensitivity of a phase measurement. For two ions, $|J = 1, m_J = 0\rangle = (|\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle)/\sqrt{2}$ can be obtained from $|\Psi_M\rangle$ by a rotation [Eqs. (2) with $\theta = \pi/2$ and $\phi = \pi/4$]. For two ions, we have $\tilde{V} = (1 + \tilde{P})/2 - \langle \tilde{J}_z\rangle^2$, so that, except for an offset and scale factor, the measured values of $\tilde{V}$ and $\tilde{P}$ are the same. Therefore we can explore the second and third methods cited in the introduction with the same experiment. Here, we cast the experiment in terms of $|\Psi_M\rangle$ and the parity operator $\tilde{P}$.

We view the experiment as performing a $\pi/2$ rotation of the state $|\Psi_M\rangle$ [Eqs. (2) with $\theta = \pi/2$] and desire to determine $\zeta = \phi$ with maximum sensitivity. To do this, we prepare the state $|\Psi_M\rangle$, perform a $\pi/2$ rotation for various values of ϕ, and measure $\tilde{P}(\phi)$, in which case $\delta\phi = \Delta\tilde{P}/|\partial\langle\tilde{P}\rangle/\partial\phi|$. The measurements are displayed in Fig. 3b. The amplitude of the observed sinusoidal oscillation is 0.845(2) rather than the theoretical maximum of 1, due primarily to imperfections in state creation. Because of these imperfections, $\delta\phi$ also depends on ϕ as shown in Fig. 3c. Ideally, we have $\delta\phi = 1/2$ ($\xi_R = 1/\sqrt{2}$), the Heisenberg limit, independent of ϕ [8]. In the experiment, we observe $\delta\phi < \delta\phi_c$ for a limited range of ϕ values. The minimal uncertainty observed is $\delta\phi = 0.59(1) < 1/\sqrt{2} = \delta\phi_c$ [$\xi_R = 0.83(1)$]. Note that the period of the oscillation of $\langle\tilde{P}\rangle$ with respect to ϕ is half (in general $1/N$ [8]) that of the period when the observable is J_z (Fig. 1b). This results in a relative increase in $|\partial\langle\tilde{O}\rangle/\partial\phi|$, which is the main reason we can find values of $\xi_R < 1$. The analog of

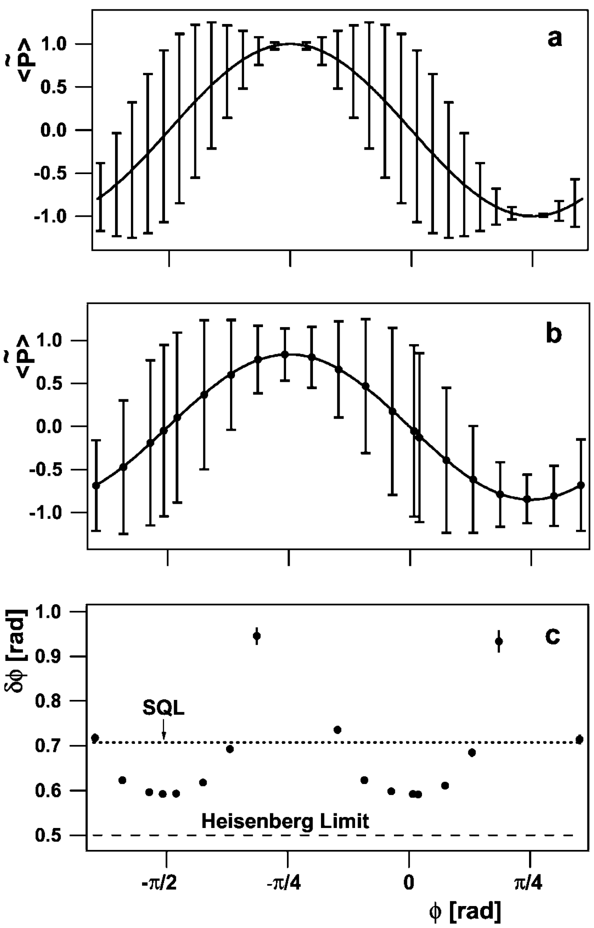

FIG. 3. In (a), we plot the theoretical value $\langle\tilde{P}(\phi)\rangle$ for an initial state $|\Psi_M\rangle = (|\downarrow\downarrow\rangle + i|\uparrow\uparrow\rangle)/\sqrt{2}$ which has been rotated through angles $\theta = \pi/2$, ϕ [Eqs. (2)]. The bars represent the variance $(\Delta\tilde{P})^2$ of the parity. In (b), we show the corresponding measured values from 10 000 experiments (experiment duration $\simeq$1 ms) per data point and a fit to the expected sinusoidal dependence. (c) The resulting phase sensitivity $\delta\phi = \Delta\tilde{P}/|\frac{\partial\langle\tilde{P}\rangle}{\partial\phi}|$, as determined from the data of (b). For the idealized experiment $\delta\phi = 1/2$, independent of ϕ [8], the Heisenberg limit. The dotted line at $1/\sqrt{2}$ represents the SQL. All figures share the same abscissa [shown in (c)].

this experiment using entangled photon pairs has been reported in Ref. [24].

These methods are of interest for improved frequency measurements [6,8]. Here, we consider use of the parity operator $\tilde{P}$ in combination with maximally entangled states [8]. In Ramsey separated-field spectroscopy [25], if the state after the first Ramsey pulse is of the form $|\Psi\rangle = [|\downarrow\rangle_1|\downarrow\rangle_2\cdots|\downarrow\rangle_N + e^{i\beta}|\uparrow\rangle_1|\uparrow\rangle_2\cdots|\uparrow\rangle_N]/\sqrt{2}$, the transition frequency between the states $|\downarrow\rangle$ and $|\uparrow\rangle$ can be determined with a precision of $\delta(\omega - \omega_0) = \delta\zeta/T = 1/(NT)$ [8]. In this expression, T is the time difference between the first and the second Ramsey pulse and $\delta(\omega - \omega_0)$ is the uncertainty in the measured frequency difference between the atoms' transition frequency ω_0 and the frequency ω of the applied radiation. Therefore, here phase sensitivity translates into frequency sensitivity through the relation $\delta\zeta = \delta(\omega - \omega_0)T$. The gain of a factor of $1/\sqrt{N}$ compared to spectroscopy of atoms in coherent states is of particular interest for precision spectroscopy, which has come close to the SQL [7,26].

To demonstrate the use of entangled states for spectroscopy, we prepare an initial state of two ions of the form $|\Psi_R\rangle \cong (|\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle)/\sqrt{2}$. The Ramsey experiment was performed on $|\downarrow\rangle \leftrightarrow |\uparrow\rangle$ transitions using stimulated Raman excitation with $\omega/2\pi$ detuned by about 10 kHz from $\omega_0/2\pi \cong 1.25$ GHz. We varied the time T between the two Ramsey pulses to change the phase difference $\zeta = (\omega - \omega_0)T$ for the experiments; that is, we determine $\omega - \omega_0$ by varying T and measuring $\tilde{P}$. In Fig. 4 the maximal gain in precision is a factor of 1.14(1) [$\xi_R = 0.88(1)$], compared to an idealized Ramsey experiment using two ions in a coherent state where preparation and detection are perfect.

In summary, we have demonstrated a fundamental increase in sensitivity of rotation angle measurement benefiting from entanglement. The reported measurements include all sources of noise (no noise subtraction)

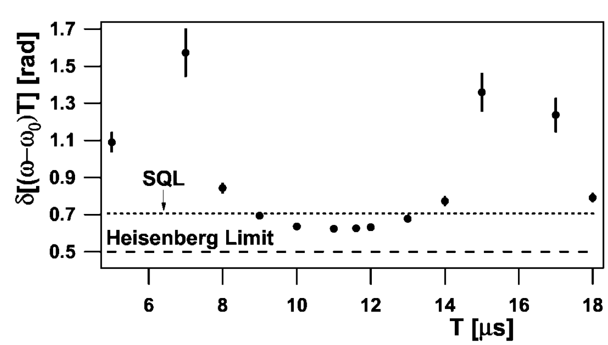

FIG. 4. Uncertainty in frequency determination in a Ramsey experiment using the input state $|\Psi_R\rangle = (|\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle)/\sqrt{2}$. We vary the time T between the Ramsey pulses for fixed frequency detuning $|\omega - \omega_0|/2\pi \cong 10$ kHz. We performed 10 000 experiments (experiment duration $\simeq$1 ms) per data point. The dotted line represents the SQL for two ions in a coherent spin state [7]. The dashed line marks the Heisenberg limit.

and demonstrate a sensitivity better than that which can be obtained without entanglement under ideal conditions. The most important limitation of our experiments is the imperfection of initial state preparation induced by heating of the ions to higher motional states [15], which reduces the degree of entanglement. Maintaining the ions close to their motional ground state should significantly reduce these effects [17]. In an application to a Ramsey spectroscopy experiment, we achieved an increased precision in frequency measurement compared to an idealized experiment using unentangled particles. This may be of significance for the construction of more precise atomic clocks.

We thank J. J. Bollinger and B. E. King for helpful comments on the manuscript. This work was supported by the U.S. National Security Agency (NSA) and the Advanced Research and Development Activity (ARDA) under Contract No. MOD-7171.00, the U.S. Office of Naval Research, and the U.S. Army Research Office. This paper is a contribution of the National Institute of Standards and Technology and is not subject to U.S. copyright.

Note in proof.—Since our submission, number-squeezed atomic states have been reported [27]. Such states are analogous to the $|J = 1, m_J = 0\rangle$ state of our experiment [3–5] and are an important step toward sub-shot-noise atom interferometry [12].

---

[1] See, for example, J. S. Bell, *Speakable and Unspeakable in Quantum Mechanics* (Cambridge University Press, Cambridge, 1987); N. D. Mermin, Phys. Today **38**, No. (4), 38 (1985).

[2] See, for example, M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, 2000).

[3] B. Yurke, Phys. Rev. Lett. **56**, 1515 (1986); B. Yurke, S. L. McCall, and J. R. Klauder, Phys. Rev. A **33**, 4033 (1986).

[4] M. Kitagawa and M. Ueda, Phys. Rev. A **47**, 5138 (1993).

[5] B. C. Sanders and G. J. Milburn, Phys. Rev. Lett. **75**, 2944 (1995).

[6] D. J. Wineland, J. J. Bollinger, W. M. Itano, F. L. Moore, and D. J. Heinzen, Phys. Rev. A **46**, R6797 (1992); D. J. Wineland, J. J. Bollinger, W. M. Itano, and D. J. Heinzen, Phys. Rev. A **50**, 67 (1994).

[7] W. M. Itano, J. C. Bergquist, J. J. Bollinger, J. M. Gilligan, D. J. Heinzen, F. L. Moore, M. G. Raizen, and D. J. Wineland, Phys. Rev. A **47**, 3554 (1993).

[8] J. J. Bollinger, W. M. Itano, D. J. Wineland, and D. J. Heinzen, Phys. Rev. A **54**, R4649 (1996).

[9] F. T. Arecci, E. Courtens, R. Gilmore, and H. Thomas, Phys. Rev. A **6**, 2211 (1972).

[10] More precisely, $\delta\zeta = \arctan(\Delta J_\perp/|\langle \boldsymbol{J} \rangle|)$ (Fig. 1a). Following convention, we use instead $\delta\zeta = \Delta J_\perp/|\langle \boldsymbol{J} \rangle|$ given by Eq. (1). In typical situations, where measurements of $\tilde{O}$ are repeated M times ($M \gg 1$), $\Delta J_\perp \rightarrow \Delta J_\perp/\sqrt{M}$ and this distinction is not important.

[11] M. J. Holland and K. Burnett, Phys. Rev. Lett. **71**, 1355 (1993).

[12] P. Bouyer and M. A. Kasevich, Phys. Rev. A **56**, R1083 (1997).

[13] T. Kim, O. Pfister, M. J. Holland, J. Noh, and J. L. Hall, Phys. Rev. A **57**, 4004 (1998).

[14] A. Sørensen, L.-M. Duan, J. I. Cirac, and P. Zoller, Nature (London) **409**, 63 (2001).

[15] Q. A. Turchette, D. Kielpinski, B. E. King, D. Leibfried, D. M. Meekhof, C. J. Myatt, M. A. Rowe, C. A. Sackett, C. S. Wood, W. M. Itano, C. Monroe, and D. J. Wineland, Phys. Rev. A **61**, 063418 (2000).

[16] C. Monroe, D. M. Meekhof, B. E. King, S. R. Jefferts, W. M. Itano, D. J. Wineland, and P. Gould, Phys. Rev. Lett. **75**, 4011 (1995).

[17] A. Sørensen and K. Mølmer, Phys. Rev. A **62**, 022311 (2000).

[18] C. A. Sackett, D. Kielpinski, B. E. King, C. Langer, V. Meyer, C. J. Myatt, M. Rowe, Q. A. Turchette, W. M. Itano, D. J. Wineland, and C. Monroe, Nature (London) **404**, 256 (2000).

[19] M. A. Rowe, D. Kielpinski, V. Meyer, C. A. Sackett, W. M. Itano, C. Monroe, and D. J. Wineland, Nature (London) **409**, 791 (2001).

[20] B. C. Sanders, Phys. Rev. A **40**, 2417 (1989).

[21] J. Hald, J. L. Sørensen, C. Schori, and E. S. Polzik, Phys. Rev. Lett. **83**, 1319 (1999).

[22] A. Kuzmich, L. Mandel, and N. P. Bigelow, Phys. Rev. Lett. **85**, 1594 (2000).

[23] In the context of Fig. 1, $|\Psi_M\rangle$ is a circle of unit radius in the plane defined by the $\hat{z}$ and $(\hat{x} - \hat{y})/\sqrt{2}$ directions and centered at the origin.

[24] A. Kuzmich and L. Mandel, Quantum Semiclass. Opt. **10**, 493 (1998).

[25] N. F. Ramsey, *Molecular Beams* (Oxford University Press, London, 1963), p. 124.

[26] G. Santarelli, Ph. Laurent, P. Lemonde, A. Clairon, A. G. Mann, S. Chang, A. N. Luiten, and C. Salomon, Phys. Rev. Lett. **82**, 4619 (1999).

[27] C. Orzel, A. K. Tuchman, M. L. Fenselau, M. Yasuda, and M. A. Kasevich, Science **291**, 2386 (2001).