Uses of entanglement in QI

Dense coding (Bennett & Wiesner 1992)

Classical: Sending more than 1 bit of information requires manipulation of more than 1 two-state particle. (Think of passing coins)

Quantum: Can do better!

1. Alice & Bob share an entangled pair of qubits, say
   \[ |\psi^+\rangle = |0\rangle_A |1\rangle_B + |1\rangle_A |0\rangle_B \]

2. Alice does 1 of 4 things to her qubit:
   i. nothing \[ |\psi^+\rangle \rightarrow |\psi^+\rangle \]
   ii. flip \[ R_x(\pi) \]
      \[ |0\rangle_A \rightarrow |1\rangle_A \quad |1\rangle_A \rightarrow |0\rangle_A \]
      \[ |\psi^+\rangle = |0\rangle_A |1\rangle_B + |1\rangle_A |0\rangle_B \]
   iii. phase \[ R_z(\pi) \]
      \[ |0\rangle_A \rightarrow |0\rangle_A \quad |1\rangle_A \rightarrow -|1\rangle_A \]
      \[ |\psi^-\rangle = |0\rangle_A |1\rangle_B - |1\rangle_A |0\rangle_B \]
   iv. flip phase \[ R_x(\pi) R_z(\pi) \]
      \[ |0\rangle_A \rightarrow |1\rangle_A \quad |1\rangle_A \rightarrow -|0\rangle_A \]
      \[ |\phi^-\rangle = |0\rangle_A |0\rangle_B - |1\rangle_A |1\rangle_B \]

4 "Bell states"
3. Alice sends her altered single qubit to Bob

4. Bob measures which Bell state he has, thus gaining 2 bits of information!!

How does Bob do this?

Controlled-NOT gate!

\[ \hat{C}N14^+ = \hat{C}N(01+11) = 01 + 11 = (0+1)1 \]
\[ \hat{C}N |\phi^+\rangle = \hat{C}N(00+11) = 00+10 = (0+1)0 \]
\[ \hat{C}N |\psi^-\rangle = \hat{C}N(11-01) = 01-11 = (0-1)1 \]
\[ \hat{C}N |\phi^-\rangle = \hat{C}N(00-11) = 00-10 = (0-1)0 \]

and rotate 1st qubit \( \frac{\pi}{2} \)

\[ R_y(\frac{\pi}{2}) (0+1) \rightarrow 0 \]
\[ R_y(\frac{\pi}{2}) (0-1) \rightarrow 1 \]

Bob:

Bob has the other, & it's enhanced! So really there are 2 qubits that are being "manipulated"!

Cheat: Yeah, Alice only send a single qubit, but Bob has the other, & it's enhanced! So really there are 2 qubits that are being "manipulated"

but notice that sent qubit carries no information!

\[ |\psi\rangle = \frac{1}{\sqrt{2}} |0\rangle + \frac{1}{\sqrt{2}} |1\rangle \]

no matter how Alice encoded.

info is in correlation \( \Rightarrow \) good for xmitting secrets
PDC experiment (Mattle et al. PRL 76 4656 (1996))

\[ \psi^+ = \psi^- \] prepared

Antisymmetric states: photons leave BS in opposite paths

\[ \psi^- = (01-10) \text{ only one!} \]

Bob observes which 2 detectors "click"

\[ D_H D_V, \text{ or } D_{H'} D_V \Rightarrow \psi^- = 01-10 \]
\[ D_H D_V, \text{ or } D_{H'} D_V \Rightarrow \psi^+ = 01+10 \]

2 clicks in any detector \[ \psi^\pm = 00\pm11 \]

Can't distinguish

2 cases \[ \log_2 3 = 1.58 \text{ bits} \]

Xmitted with a single 2L particle!

Scaling: Max. Entangled M-level system

\[ \log_2 M \text{ bits sent} \]
\[ 2 \log_2 M \text{ bits received} \]

Still gain only factor of 2
Quantum Teleportation (Bennett, Brassard, Cribbs, \textit{et al.}, 1993)

Alice has qubit \( \alpha |0\rangle + \beta |1\rangle \), she wants to convey the quantum information \((\alpha, \beta)\) to Bob without measuring \(\alpha, \beta\)!

- just give Bob the qubit carefully, who looks it!
  (trivial)

- Alice prepares her qubit many times and reconstructs \((\alpha, \beta)\):
  \[
  P_0 = |\alpha|^2 \\
  P_1 = 1 - P_0
  \]

Hadamard \( \hat{H}^4 = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) (|0\rangle - |1\rangle) \)

\[
\begin{align*}
P_0' &= \frac{1 - |\beta|^2}{2} = \frac{1}{2} - \text{Re} \alpha \beta^* \\
\end{align*}

then tells Bob over classical channel

But qubits may be expensive! May only have one.

Assume A & B share a maximally-entangled pair of qubits.
Alice can then "teleport" qubit to Bob.

![EPR pair diagram]

Comments
- Entanglement likely much more expensive than repeated preparation of \( \alpha |0\rangle + \beta |1\rangle \)!
- BUT, communication channel may be closed, or Alice’s mystery qubit e.g. may be in a difficult form to just send to Bob - atoms vs. photons
Quantum Teleportation Protocol

\[ |00\rangle - |11\rangle \quad |\psi^-\rangle + \]
\[ \langle 0|0\rangle - |11\rangle \quad |\phi^-\rangle + \]
\[ \langle 0|1\rangle + \langle 1|0\rangle \quad |\phi^+\rangle + \]
\[ \langle 0|0\rangle + \langle 1|1\rangle \quad |\phi^+\rangle \]

Just another page


\[ \begin{align*}
\langle 0|0\rangle + \langle 1|0\rangle + \langle 1|1\rangle + \langle 1|0\rangle &= \langle 0|0\rangle \\
\end{align*} \]


Bell States

\[ \begin{align*}
\langle 0| - \langle 1| & = |\psi^-\rangle \\
\langle 0| + \langle 1| & = |\phi^-\rangle \\
\langle 0| - \langle 1| & = |\psi^+\rangle \\
\langle 0| + \langle 1| & = |\phi^+\rangle
\end{align*} \]
3. Alice phones Bob, tells him which Bell state she got (2 classical bits)

4. Bob does the following Rotation, depending on Alice's result:
   - if $\phi_{AA}^+ \rightarrow \hat{1}$ (nothing)
   - if $\phi_{AA}^+ \rightarrow R_x(\pi)$ (Flip qubit)
   - if $\phi_{AA}^- \rightarrow R_z(\pi)$ (\pi-phase gate)
   - if $\phi_{AA}^- \rightarrow R_x(\pi)R_z(\pi)$ (Flip \& \pi-phase gate)

Bobs qubit: $\psi_B = \alpha|0\rangle + \beta|1\rangle$
reconstructed!

Comment: Teleportation refers to the information, not the actual physical qubit!

photon qubit $\rightarrow$ atom qubit
Kirk $\rightarrow$ Spock
how does info get from Alice to Bob? Classical bits are random! Entanglement?
no-cloning OK. $|\Theta\rangle$ destroyed before Bob recreates.

Fidelity of teleportation (how to compare 2 quantum states?)

pure-pure: $F = |\langle \Theta | \psi_B \rangle|^2$ overlap between initial state $|\Theta\rangle$ and Bob's state $|\psi_B\rangle$.

if Bob gets a mixed state

pure-mixed: $F = \langle \Theta | \rho_B | \Theta \rangle$

(if Alice sends a mixed state $\rho_A$)

mixed-mixed: $F = \text{Tr}(\sqrt{\rho_A} \rho_B \sqrt{\rho_A})$ ... let's shake w/ $|\Theta\rangle$!)
What's one interesting level of fidelity??

1. Bob could simply guess what $|\theta\rangle$ is (assume $|\theta\rangle$ is uniformly distributed on Bloch sphere)

$$F = \langle \theta | \psi_{\text{guess}} \rangle^2$$

E.g. say $\psi_{\text{guess}} = |10\rangle$ always

Then $F = |\langle \theta | 10 \rangle|^2 = \frac{1 + \cos \theta}{2} = \frac{1}{2}$

2. Better: Alice measures her unknown qubit along $\hat{z}$, then tells Bob (classically) her result. Bob then reconstructs his qubit in the state Alice measured:

Alice's state is (Bob's reconstructed state) $|\psi\rangle = \frac{1 + \cos \theta}{2} |0\rangle + \frac{1 + \cos \theta}{2} |1\rangle e^{i\phi}$

$$F = \langle \theta | 0 \rangle \langle 0 | \theta \rangle = \langle \frac{1 + \cos \theta}{2} |0\rangle^2 + \langle \frac{1 + \cos \theta}{2} |1\rangle^2$$

= $\langle \frac{1}{2} + \frac{1}{2} \cos^2 \theta \rangle$

= $\left\langle \frac{1}{2} + \frac{1}{2} \cos^2 \theta \right\rangle$

= $\frac{2}{3}$

$$F = \frac{2}{3} \quad \Rightarrow \quad \frac{1}{2} \text{ because A&B have gained info about } |\theta\rangle$$
Experimental quantum teleportation

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Experimental Realization of Teleporting an Unknown Pure Quantum State via Dual Classical and Einstein-Podolsky-Rosen Channels

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Unconditional Quantum Teleportation

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Quantum teleportation of optical coherent states was demonstrated experimentally using squeezed-state entanglement. The quantum nature of the achieved teleportation was verified by the experimentally determined fidelity $F = 0.58 \pm 0.02$, which describes the match between input and output states. A fidelity greater than 0.5 is not possible for coherent states without the use of entanglement. This is the first realization of unconditional quantum teleportation where every state entering the device is actually teleported.

Quantum teleportation is the disembodied transport of an unknown quantum state from one place to another (1). All protocols for subsystems of infinite-dimensional systems where the above advantages can be put to use. Typically, a definite, definite state is used as an output $\beta_{\text{out}}$ that closely mimics the original unknown input.

In our scheme, a third party, Victor (the verifier), prepares an initial input in the form of a coherent state of the electromagnetic field $|\psi_{1}\rangle$, which he then passes to Alice for teleportation. Likewise, the teleported field that emerges from Bob's sending station is interrogated by Victor to verify that teleportation has actually taken place: At this stage, Victor records the amplitude and variance of the field generated by Bob, and is thereby able to assess the "quality" of the teleportation protocol. This is done by determining the overlap between input and output as given by the fidelity $F = \langle \psi_{1}|\beta_{\text{out}}|\psi_{1}\rangle$. As discussed below, for the teleportation of coherent states, $F_{c} = 0.5$ sets a boundary for entrance into the quantum domain in the sense that Alice and