Differential Eqns

1. Solve the below ordinary differential equations, where C is a positive constant. Include and label any integration constants.
   
   (a) \( \frac{dy}{dx} = Cy \)
   
   (b) \( \frac{d^2y}{dx^2} = Cy \)
   
   (c) \( \frac{d^2y}{dx^2} = -Cy \)
   
   (d) \( \frac{d^2y}{dx^2} = Cx \)

2. Provide the form of \( f(x,t) \) in the below partial differential equations, where C is a positive constant. You don’t have enough information to solve it, but simply show how the function \( f \) must depend upon \( x \) and \( t \).
   
   (a) \( \frac{\partial^2 f(x,t)}{\partial t^2} = C \frac{\partial^2 f(x,t)}{\partial x^2} \)

   (b) \( \frac{\partial f}{\partial t} = C \frac{\partial^2 f}{\partial x^2} \)

Complex Variables

3. Simplify the below expressions into a sum of real and imaginary parts \( a+bi \)
   
   (a) \( i^i \)

   (b) \( (-1)^{1/4} \)

   (c) \( \log(i) \) (this is a natural log to base \( e \), also known as \( \ln \))

4. Use Euler’s formula to derive the below trig identities
   
   (a) \( \cos(a+b) = \cos a \cos b – \sin a \sin b \)

   (b) \( \cos^2 x = (1+\cos 2x)/2 \)

   (c) \( \cosh(x) = \cos(ix) \)
Fourier Transforms

5. Find the Fourier transform \( F(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt \) for a square pulse of duration \( \tau \):

\[
f(t) = 1 \text{ (for } 0 < t < \tau); \quad f(t) = 0 \text{ otherwise.}
\]

What is the spectral bandwidth of \( F(\omega) \)?

Probability

6. The lifetime of a lightbulb is commonly modeled as an exponentially-distributed random variable \( t \) with probability distribution function

\[
f(t) = \frac{1}{\tau} e^{-t/\tau} \quad \text{for } t \geq 0,
\]

where \( \tau \) is a parameter in the distribution. Find the mean and standard deviation of the random variable \( t \).

Linear Algebra

7. Find the eigenvalues and eigenvectors of the 3x3 matrix

\[
\begin{bmatrix}
3 & 0 & -2 \\
0 & 8 & 2 \\
0 & -3 & 1
\end{bmatrix}
\]